DERIVATIVE TASKS PROPOSED BY TEACHERS IN TRAINING

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The tasks are fundamental elements in the process of teaching and learning of Mathematics and consequently, the design and selection of these. The present work focuses on characterizing a set of tasks proposed by teachers in training and focused on the derivative of a function at one point, as part of a broader investigation in which we address the meaning that teachers attribute to this topic. We employ the method of content analysis provided with a system of categories that allow us to analyze revealed elements of the meaning of the derivative as well as the cognitive demand encouraged for each of the tasks. We noticed a clear predominance of tasks with low cognitive demand in which the most involved content is the calculation of maxima and minima of a function.

INTRODUCTION

The relationship between the type of homework tasks that students do and the mathematics they learn has been a topic of research for many years (Breen & O'Shea, 2010). Several studies claim that what students learn is highly determined by the tasks assigned by teachers (Sullivan, Clarke, & Clarke, 2013). Concretely, tasks transmit messages about what mathematics is and what it involves to know them, that is, its meaning. Moreover, it is considered that it is through the tasks that students are really given learning opportunities (Anthony & Walshaw, 2009). Some authors even think that posing tasks that invite the student to think for himself is the main stimulus for learning, above any other action in the classroom, (Sullivan, Clarke and Clarke, 2013). Hence, the design and selection of tasks are essential for effective teaching (Watson et al., 2014).

Due to its relevance, in recent years, there has been an increasing interest in addressing investigations about school tasks (e.g. Lithner, 2017; Bobis, et al., 2019). Other aspects also show its relevance: for example, in 2003, at the annual meeting of the international group for the Psychology of Mathematics Education (PME), the design and use of tasks were identified as the main topics of the research reports. As well, in 2008, the International Congress of Mathematical Education (ICME) organized a Topic Study Group (TSG), named Research and development in task design and analysis, and even some journals, such as the Journal of Mathematics Teacher Education (JMTE), have devoted a special issue to this topic.

Therefore, we believe that teachers should be able to pose tasks that promote appropriate learning of their students (Lee, Lee, & Park, 2016). Both the design and

2021. In Inprasitha, M, Changsri, N., & Boonsena, N. (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 185-192. Khon Kaen, Thailand: PME.

the selection of tasks are influenced by the teacher's goals, as well as by his knowledge and beliefs about mathematics (Sullivan, Clarke, & Clarke, 2013). Therefore, as part of a broader investigation in which we address the meaning of the derivative expressed by math teachers, we focus the present work on characterizing the derivative tasks proposed by teachers in training. In this way, not only we approach the future teachers' perception of the derivative, but also, we can also analyze the relevance of these tasks.

BACKGROUND

We understand ‘task’ in terms of Watson et al. (2014) “to mean a wider range of ‘things to do’ than this, and include repetitive exercises, constructing objects, exemplifying definitions, solving single-stage and multi-stage problems, deciding between two possibilities, or carrying out an experiment or investigation” (p. 9-10). Indeed, a task is anything that a teacher uses to ask students to do something.

Different models and approaches have been used for the analysis of mathematics tasks. One widely used has to do with the level of cognitive demand of the task, proposed by Stein, Grover, and Henningsen (1996), and used in many investigations (e.g., Cai, Moyer, Nie, & Wang, 2009; Tekkumru-Kisa, Stein, & Schunn, 2015). In our work we also consider the four levels of cognitive demand raised there. However, since our goal is also related to the meaning of the derivative for math teachers, we extend the analysis of the tasks with a set of categories proposed by Moreno and Ramírez (2016) which have been already used in Vargas, Fernández-Plaza, and Ruiz-Hidalgo (2018). These categories can be organized in two groups:

- Mathematical content and its meaning: considering the theoretical framework based on the meaning of a school mathematical concept developed by Rico (2013), we analyze some elements that make up the meaning of the derivative: the content, the representation systems, the transformation of representation systems that are requested, the context, the situation and the type of function involved.
- Learning or cognitive aspects: regarding this aspect we analyze (a) the demand (Stein et al., 1996); and (b) the mathematical ability fostered by the task.

METHOD

We conduct qualitative research of descriptive nature, which was carried out with 55 Mathematics teachers in training in Spain. At the time of data collection, they were studying for the University Master's Degree in Secondary Education at the University of Granada, intended for graduate students with different academic backgrounds (Mathematics, Engineering, Physics, among others) who wish to access to secondary teacher career. In this way, every considered future teacher has passed at least two Calculus courses in their training.

A survey composed of three questions was used for the data collection. From the questions, we present here the one in which teachers in training were asked to propose a task that was resolved involving the derivative. Through a content analysis, we proceeded to study each of the proposed questions according to each of the following categories.

System of Categories

In the group of Mathematics meanings, we considered the following categories:

1. *Content*: the mathematical content addressed in each task.
2. *Representation systems*: we pay attention to the different representation systems that appear in the statement of the task. These can be: verbal, graphic, numerical, symbolic and / or tabular.
3. *Types of transformations*: under the line of representation systems and following Duval (2006), we analyze whether the proposed task encompasses in its resolution transformations within the same system (treatments) or requires translations from one system to another (conversion).
4. *Situation*: we identify the PISA situation (OECD, 2016) in which the proposed tasks are presented: personal, occupational, societal, or scientific.
5. *Context*: based on our theoretical framework, we take into account the different mathematical contexts or functions to which the derivative attends in each one of the tasks.

From the group of cognitive aspects, we consider:

1. Cognitive demand: To analyze this aspect we use the taxonomy of Stein et al. (1996), in which four types of tasks are considered, according to cognitive demand. The characterization of these can be seen in Table 1.

Cognitive demand	Description
Memorization	Regarding those tasks that ask the student to remember facts, rules or definitions. The answers imply an exact and memorized reproduction. No type of procedure is used.
Procedures without connections	The purpose of this kind of task is to apply some algorithm to solve a problem. It is more about applying than understanding. These tasks are characterized by not requiring explanations as well as there is no ambiguity about what to do and how to do it.
Procedures with connections	Although these tasks have a procedure to be solved, their intention goes beyond the process itself, trying to develop deeper levels of understanding about mathematical concepts and ideas. Its main feature is that they are not tasks that can be solved only by knowing the algorithm, but they require some extra effort.
Doing Mathematics	These are the tasks with the highest cognitive demand, since they require non-algorithmic thinking and the solution path is not predetermined. They require a true understanding of the concepts, processes, properties and the establishment of relationships among mathematics concepts.

Table 1: Taxonomy of Stein et al. (1996)

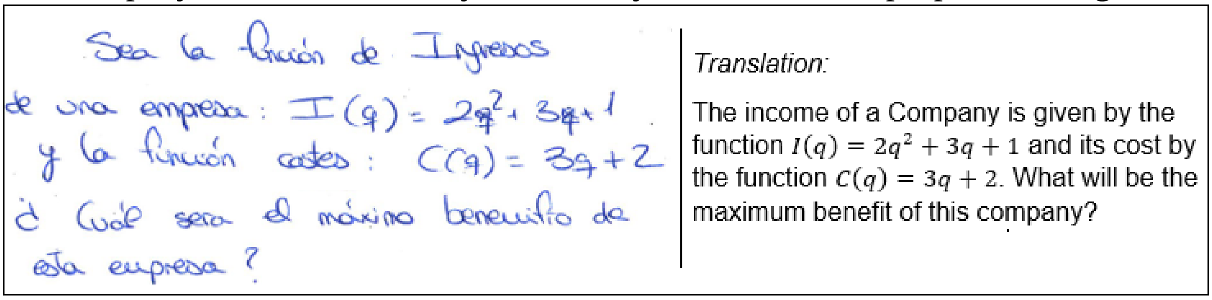
2. Mathematical capability: We adopt the seven Fundamental Mathematical Capabilities from PISA framework (OECD, 2016): communication; mathematising; representation; reasoning and argument; devising strategies for solving problems; using symbolic, formal and technical language and operations; and using mathematical tools.

RESULTS

One out of the 55 participants did not write any task, other six teachers in training drew surfaces in which derivatives should be used, but they did not actually pose a task. Although they were only asked to write a task that was solved through the derivative, some of them posed two or three, so a total of 52 tasks were analyzed.

At first glance, we detected that nine out of the 52 tasks have no solution as they were written, since either the necessary data were not presented, or the data were not consistent. Despite this, these nine tasks took part of the analysis, considering the intention with which they were posed.

To exemplify, we show the analysis we carry out for the task proposed in Figure 1.


Sea la función de Ingresos de una empresa: $I(q) = 2q^2 + 3q + 1$ y la función costes: $C(q) = 3q + 2$ ¿Cuál será el máximo beneficio de esta empresa?

Translation:

The income of a Company is given by the function $I(q) = 2q^2 + 3q + 1$ and its cost by the function $C(q) = 3q + 2$. What will be the maximum benefit of this company?

Figure 1: Example of task posed

The first step of the analysis was to identify optimization as the content that is addressed in the task (Figure 1). This task is contextualized in an occupational situation, in a context of applications of the derivative. Regarding the representation system, both verbal and symbolic are used, where the management required is a symbolic treatment of the given function.

Regarding the demand of the task, the resolution of a plain problem is requested, in which there is already defined the function that models the situation and could even be solved without using the notion of derivative (notice the vertex of a quadratic function). Therefore, it could be solved using procedures without connections. In fact, students usually learn to solve automatically these types of tasks, without really needing the context to find the correct answer. Finally, the capacity that it fosters is classified as the use of operations and symbolic language.

In the following, we show the results obtained after the analysis of the 52 tasks, for each of the categories considered.

Regarding the *mathematical content* addressed in the tasks, problem solving predominated (26), mainly optimization, and particle speed and acceleration; followed by tasks in which it is determined the extremes values of a function and analyze its monotony (16).Other tasks also addressed: derivation rules (2), differential equations (1), calculation of limits (2), tangent and normal lines (3); and others (2). We have included the category 'others' for those tasks that did not really address any content of the derivative, as is the case of a participant who stated: “Indicate the intervals of increasing and decreasing for the function shown [...]”. The solution does not require the notion of derivative. In this regard, seven out of the 52 tasks that address relative extremes and monotony do by using quadratic functions, for instance $f(x) = x^2$, so that knowing the sketch of such a graph could determine the extrema without involving the notion of derivative.

A plain function was involved in most tasks (in 33 tasks), mainly polynomials of grade 2 or 3. Four of the tasks slightly suggest more complex functions, and no specific function is proposed in the remaining 15 tasks, but in some cases the solver is who may determine the function modelling the situation to answer the task.

The *representation system* used in the statement and the *types of transformations* that are requested of these can be seen in Table 2. The only representation systems that emerged were verbal and symbolic, or both. The management of representation systems mainly deals with a symbolic treatment. In the case of conversion, it refers in all cases to the translation from the verbal to the symbolic system.

Representation system	Types of transformations	Frequency
Symbolic	Treatments	20
Verbal	Conversion	16
Verbal y symbolic	Treatments	16

Table 2: Representation systems used in tasks and types of transformations

Scientific *situations* predominate, specifically intra-mathematical situations. A half of the tasks analyzed were contextualized in a strictly mathematical situation (26), the second half were categorized in occupational situations (12), mainly issues of business benefits), physics (9, speed and acceleration of bodies), and personal (5). The *context* within which the tasks were proposed was mainly applied (29), followed by geometric (16) and a few within an algebraic-numerical context (7).

An interesting aspect of the tasks is the *cognitive demand* of each of them. Table 3 shows what is requested in the tasks and the related demand.

Cognitive demand		Frequency
Procedures without connections	Direct Calculation	6
	Indirect Calculation	19
	Problem solving	15
Procedures with connections	Identifying	1
	Problem solving	11

Table 3: Cognitive demand of the proposed tasks

We notice that the tasks mainly require procedures without connections to be solved. The only task of identification is not really about derivation. In the same way, the most of the 11 problems considered to have a higher level of demand (procedures with connections) correspond to tasks that had no solution. However, according to the intention of the proposal and the amount of data that would be required for its solution, the demand is different from others problems such as that shown in Figure 1 that can be solved more mechanically. In addition, in these 11 problems the situation plays an important role, since the task must be interpreted in order to find a modelling function.

In a similar way to the cognitive demand of each task, it is possible to analyze the mathematical capacity that each one of them promotes. The most encouraged capacity has to do with calculations and symbolic language (in 50 tasks), regardless of whether tasks are found in different contexts and situations. Since there are so many tasks in the form of contextualized problems, we can also say that the design of strategies to solve problems is promoted and in some of them, mathematising (11).

DISCUSSION

The goal of this work was to characterize tasks posed by teachers in training, for the topic of derivative. The analysis showed that the tasks proposed mainly addressed the content of finding maxima and minima, as well as problem solving. Although many future teachers submitted task in an applied context, the most of them were placed in mathematical situations. We also identified that the tasks were formulated using mainly verbal and symbolic representation systems, and what is requested is essentially a procedure without connections that only requires symbolic transformations (treatments).

This is a noteworthy result since it has been found that tasks should lead to more rigorous ways of thinking. In fact, it has been determined that students learn best when they attend lesson in which they maintain a high level of cognitive demand (Kessler, Stein, & Shunn, 2015), i.e., the tasks proposed must demand a procedure with connections or doing Mathematics. However, we realize that teachers in training essentially propose tasks that promote the handling of quick procedures. Thus, according to Sangwin (2003), it is clear that routine tasks that solve without the use of superior skills predominate. We believe that, regardless of the context, for a task to be worthwhile, it must be interesting and provide a level of challenge that invites reflection and hard work (Cai, Moyer, Nie and Wang, 2009). Even though students prefer simple tasks, they consider that they learn more with demanding tasks (Sullivan, Clarke and Clarke, 2013).

The Stein et al. 's (1996) claim about the tendency of the teacher to reduce the level of potential demand of the task is related to the result of this paper. Although an effort is made to contextualize the task, they are finally very simple problems that are solved by applying a routine procedure. Charalambous (2008) argues that a factor in this is the teacher knowledge. Also, we think the posing of tasks is also related to the content is understood, i.e. to the meaning given to the notion of derivative.

A synthesis of the tasks analyzed shows that the derivative is perceived as an algebraic tool to determine the extremes of a function. Certainly, this is a fairly limited view of what this concept encompasses. We believe that the results obtained can be used as input in the training of teachers in order to enrich the meaning of the concept of derivative, and that in this way teachers can in the future select and design varied tasks that enrich the meaning of this concept in their students.

Acknowledgments

This study was developed within the framework of the National I+D+I Plan through project PCG2018-095765-B-100, funded by the Spanish Ministry of Science and Technology. We also thank the University of Costa Rica for the given scholarship to the author Vargas, which has allowed her to work on this research.

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