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STATISTICAL SIGNIFICANCE OF A WEAK SPECTRAL PEAK USING FILTERING TECHNIQUES: A CASE STUDY

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ABSTRACT

Filtering techniques are applied to geopotential series at the 850, 700 and 300 hPa levels at Howard Base, Panama to determine if the weak spectral peak near a frequency of 0.25/day can be considered real or not. An ARMA bandpass filter and a pole cancelling method are applied to the data. Statistical tests suitable to filtered series are applied and it is determined that the hypothesis that the peaks are real is not supported.

RESUMEN

Se aplican técnicas de filtrado a series de geopotencial en los niveles de 850, 700 y 300 hPa de Howard Base, Panamá para determinar si un pico espectral débil cerca de la frecuencia de 0.25/día puede ser considerado real o no. Los filtros aplicados a los datos son un filtro pasa-banda ARMA y un filtro para cancelar polos. Los ensayos estadísticos utilizados son aquellos apropiados a series filtradas y no respaldan la hipótesis que el pico es real.

1. INTRODUCTION

In a recent paper (Soley 1987), the author presented some statistical significance tests, which are useful in spectral analysis of time series. The tests were applied to meteorological time series from several stations in the Western Caribbean. One particular example dealt with the geopotential at the 850, 700 and 300 hPa levels at Howard Base during the period June 1 to August 31, 1979. The spectra at each of the three levels, showed a weak but statistically significant peak near a frequency of 0.25/day, and it was shown that the appropriate autoregressive-moving average ARMA (p,q) models to represent the three series, were pure autoregressive models of order three AR(3). The frequency range corresponding to periods between 3 to 5 days, is specially interesting since it is known that synoptic scale wave-like disturbances exist in it (Palmer 1952; Riehl 1954; Yanai et al., 1968; Carlson 1969 a, b; Wallace and Chang, 1969; Julian 1971; Reed and Recker, 1971; Reed et al., 1977; Nitta et al., 1985). Some of our data showed peaks with statistical significance levels against AR(1) models near 0.05 in one or two of the three levels analyzed. It was suggested to the author during informal discussions that perhaps filtering the data, and thus improving the signal to noise ratio, might help in deciding which of the peaks in the threshold boundary were real or not.

The purpose of this paper is to present the results of applying filtering techniques to the data and running statistical tests on the filtered series. An ARMA band pass filter designed to resemble a Butterworth filter and a pole cancelling method are applied to the data. ARMA models are fitted when necessary and statistical tests suitable to filtered series are applied.

2. METHODS

The length of the series runs usually from 3 to 5 months of daily data in the second half of the year, which is the period when the wave-like disturbances are known to exist. If a non-recursive finite impulse response (FIR) filter is to be used, its length must be short compared to the total

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length of the series to be filtered. The length must be such that the data lost because of the impossibility of calculating the filter's output near the ends is kept within the generally accepted value of less than 20%. In our case thar means that the maximun number of filter elements must be kept between 9 and 16. It must be remembered that Hamming (1977) warns against the common practice of supplying a sequence of zero values beyond one or both ends of the data, so that as many output values can be computed as there were original ones. The author tried to use traditional methods of FIR filter design (Rabiner 1971) and minimax designs (Rabiner et al., 1970; Peled and Liu, 1976) without success because it proved to be impossible to obtain suitable frequency response characteristics within the length constraints.

The author chosed then to use **ARMA** filters that have the advantage of reducing the data loss due to end effects. The possibility that a recursive filter might become unstable is prevented by making sure that the poles of the transfer function are all within the unit circle.

The following two sections review briefly some properties of **ARMA** processes and filters.

a. ARMA process

Following Box and Jenkins (1976), an ARMA (p, q) is defined by the linear difference equation

$$x_n = \sum_{k=1}^{p} a_k * x_{n-k} + n_n - \sum_{l=1}^{q} b_l * n_{n-l} \quad (1)$$

Where the a_k are the p autoregressive coefficients and the b_l the q moving average coefficients. Notice that they have the opposite sign of the coefficients used by Kay and Marple (1981). n_n is the white noise driving sequence of zero mean and variance σ^2. Let A(z) and B(z) be the zeta transforms of the autoregressive and the moving average branch, respectively; then

$$A(z) = 1 - a_1 z^{-1} - a_2 z^{-2} \ldots\ldots - a_p z^{-p} =$$

$$= 1 - \sum_{k=1}^{p} a_k z^{-k}, \text{ and} \quad (2)$$

$$B(z) = 1 - b_1 z^{-1} - b_2 z^{-2} \ldots\ldots - b_q z^{-q} =$$

$$= 1 - \sum_{l=1}^{q} b_l z^{-l} \quad (3)$$

Factoring the z transforms, one obtains

$$A(z) = \prod_{k=1}^{p} (1 - F_k z^{-1}), \text{ and} \quad (4)$$

$$B(z) = \prod_{l=1}^{q} (1 - G_l z^{-1}) \quad (5)$$

where F_k and G_l are the roots of the polinomials. The condition for stationarity and invertability demand that the roots should lie within the unit circle in the complex plane.

The power spectrum is known to be

$$p(f) = \Upsilon \sigma^2 \, |B(z)|^2 / |A(z)|^2 \quad (6)$$

with $z = \exp(j\, 2\pi f \Upsilon)$, Υ the sampling interval and $|f| \leqslant f_N = 1/2\Upsilon$, f_N being the Nyquist frequency. For a_k and b_l real, the roots of A(z) and B(z) lie in the real axis or occur in complex conjugate pairs.

The numerator and denominator have terms all of the form $|1 - H z^{-1}|^2$ which can be rewritten as $|z^{-1}(z - H)|^2 = |z - H|^2$, since $|z^{-1}|^2 = 1$ when z is evaluated in the unit circle. The geometrical interpretation of the last form is the distance squared between the complex point H and the position in the unit circle corresponding to frecuency f. Then the power spectrum can be rewritten as follows (Rader and Gold, 1967)

$$p(f) = \Upsilon \sigma^2 \frac{R_1^2 R_2^2 \ldots\ldots R_q^2}{P_1^2 P_2^2 \ldots\ldots P_p^2} \quad (7)$$

Here R_l is the distance from the unit circle at an angle $2\pi f \cdot \Upsilon$ to the l th root of the moving average branch, and P_k the distance to the k th root of the autoregressive branch. The zeroes of the moving average branch are the zeroes of the system function and the zeroes of the autoregressive branch the poles of the system function.

b. ARMA filter

Similary, an ARMA (P, Q) filter is defined by

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \ldots\ldots + A_P y_{t-P} + x_t - B_1 x_{t-1}$$

$$-B_2 x_{t-2} - \ldots\ldots\ldots - B_Q x_{t-Q} \qquad (8)$$

Where x_t and y_t are the input and output, respectively, at time t. The filter's system function H(z) is

$$H(z) = Y(z)/X(z) = \frac{(1 - B_1 z^{-1} \ldots\ldots - B_Q z^{-Q}}{(1 - A_1 z^{-1} \ldots\ldots - A_P z^{-P})} \qquad (9)$$

The system function may be factored as

$$H(z) = \bigcap_{L=1}^{Q} (1 - D_L z^{-1}) \Big/ \bigcap_{K=1}^{P} (1 - C_K z^{-1}) \qquad (10)$$

If the series x_t is an ARMA (p, q) process, the filter's output is related to the white noise driving series by

$$Y(z) = \frac{\bigcap_{L=1}^{Q} (1 - D_L z^{-1}) \quad \bigcap_{1=1}^{q} (1 - G_1 z^{-1})}{\bigcap_{K=1}^{P} (1 - C_K z^{-1}) \quad \bigcap_{k=1}^{p} (1 - F_k z^{-1})} N(z) \qquad (11)$$

where N(z) is the zeta transform of the white noise series.

From Ec. 11 one can see that a pole (zero) in H(z) may be cancelled by a zero (pole) in the filter's system function. Thus, an AR (1) process can be whitened by an MA (1) filter if $D_1 = F_1$ In practice, since the coefficients are not exactly known but can only be estimated, it is impossible to obtain perfect cancellation.

It is known that Butterworth filters of high orders have frequency responses that resemble brickwall characteristics. The ban pass characteristics desired to filter the portion of the spectra near .20/day are not very stringent and in our case are satisfied by a Butterworth filter of order two. An order two low pass Butterworth filter is defined by the transfer function.

$$H(s) = 1/(s^2 + \sqrt{2}\, s + 1) \qquad (12)$$

Through the transformation (Rader and Gold, 1967) $s = (z^2 - 2z \cos\phi T + 1)/(z^2 - 1)$, the low pass filter can be transformed into a band pass filter where ϕ is the angular frequency at the center of the pass band. Notice that the resulting zeta transfer function corresponds to an ARMA (4, 4) for which the MA branch has the form $(1 - z^{-2})^2$, that is, with two second order zeroes at $\pm$ 1. We have experimented with ARMA (4,4) filters similar to the above with excellent frequency characteristics and great ease of design. For example, placing the zeroes of the AR branch at the points in polar coordinates given by (.8584, $\pm$ 62°) and (.8425, $\pm$ 81°), one obtains a band pass filter centered at 0.2 normalized frequency and frequency response shown in Fig. 1 The MA coefficients are $b_1 = 0$, $b_2 = 2$, $b_3 = 0$ and $b_4 = -1$, whereas the AR coefficients are $a_1 = 1.0695$, $a_2 = -1.6590$, $a_3 = .76623$ and $a_4 = -.52293$. Notice that the zeroes on the unit circle make the filter non invertible. Removing the zeroes to $\pm$ 0.9 causes only a slight change on the frequency response (Fig. 1). It must be stressed that this filters are not strictly Butterworth because they are designed by judiciously placing the poles in the complex plane, and then the AR coefficients will not necessarily satisfy the relation implied by Ec. 12.

c. Statistical significance test of the output power

Following Olberg (1972, 1982) let q be the ratio of the average power of the filtered data and the variance of the filtered model $\sigma^2{}_{FM}$

$$q = \sum_{j=1}^{N} x_f^2 \,/(N\, \sigma^2{}_{FM}) \qquad (13)$$

The variance of the filtered model is obtained from the cross product of the filter autocorrelation coefficients W (j) with the model autocorrelation coefficients Km (j).

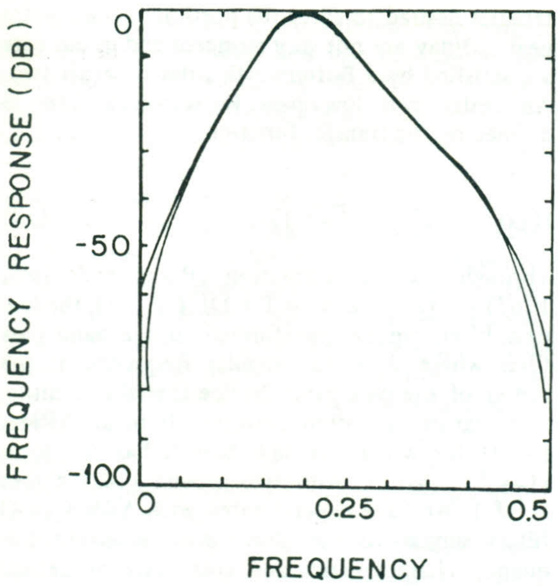

Fig. 1. ARMA (4,4) band pass filter frequency response. The lower curve is the response with zeroes in the unit circle and the upper curve the response with the zeroes removed from the unit circle.

$$\sigma^2_{FM} = \sigma^2_{M} \sum_{j=-2m}^{2m} W(j)\, K_M(j) \tag{14}$$

where σ^2_M is the variance of the model.

Olberg (1972) has shown that q is distributed asymptotically normal with expected value 1 and variance

$$\mathrm{Var}(q) = \frac{2}{N} \sum_{j=-N+1}^{N-1} (1\ \mathrm{abs}(j)/N)\, K^2_{FM}(j) \tag{15}$$

where the K_{FM} (j) are the autocorrelation coefficients of the filtered model,

$$K_{FM}(j) = (\sigma^2_M / \sigma^2_{FM}) \sum_{k=-2m}^{2m} W(k)\, K_M(\mathrm{abs}(k-j)) \tag{16}$$

In the special case that the model is white noise, the autocorrelation series K_M (j) is the delta series and therefore

$$K_{FM}(j) = W(j) \tag{17}$$

3. DATA AND RESULTS

a. Data

The data used to illustrate the filtering procedures are the zonal wind component and geopotential at the 850 hPa level at Swan Island Station during the period June 1 to October 31, 1970. The zonal wind series has the usual red noise type spectra found in most meterological time series (Julian 1971; Madden and Julian, 1971; Jones 1974; Soley 1987), and follows an AR(1) process with $a_1 = 0.428$, as can be verified with the procedures in Soley (1987). The Blackman-Tukey and AR(1) estimates of the spectral power density are shown in Fig. 2. The spectra shows apparently significant peaks at frequencies of 0.1 and 0.2/day, but when tested against the AR(1) model their significant levels are .20 and .09, respectively.

In the case of the geopotential series, the autocorrelation and partial autocorrelation functions (Fig. 3) suggest an AR process of order four, a choice that was confirmed by the Akaike Information Criteria (AIC) and Parzen's Criteria Autoregressive Transfer (CAT) function (Kay and Marple 1981). The Blackman-Tukey power spectral density estimate is shown in Fig. 4. The peak near frequency 0.2/day has a significance level of .0055 when tested against an AR(1) model. In spite of its relatively high significance level, the author considers this peak as doubtful because the fourth partial autocorrelation coefficient is just outside the two standard deviations threshold and although not shown, the Generalized Partial Autocorrelation (GPAC) function (Woodward and Gray 1981) does not follow the expected behaviour. The Burg estimates for the AR coefficient are $a_1 = 0.74579$, $a_2 = -0;12459$, $a_3 = -0.02772$ and $a_4 = 0.16942$. The poles of the transfer function are at points in the complex plane giving in polar coordinates by (0.8423, 0°), (0.4969, 180°) and (0.6363, ± 71.66°).

To check the results when filtering the data, two series closely resembling the above ones

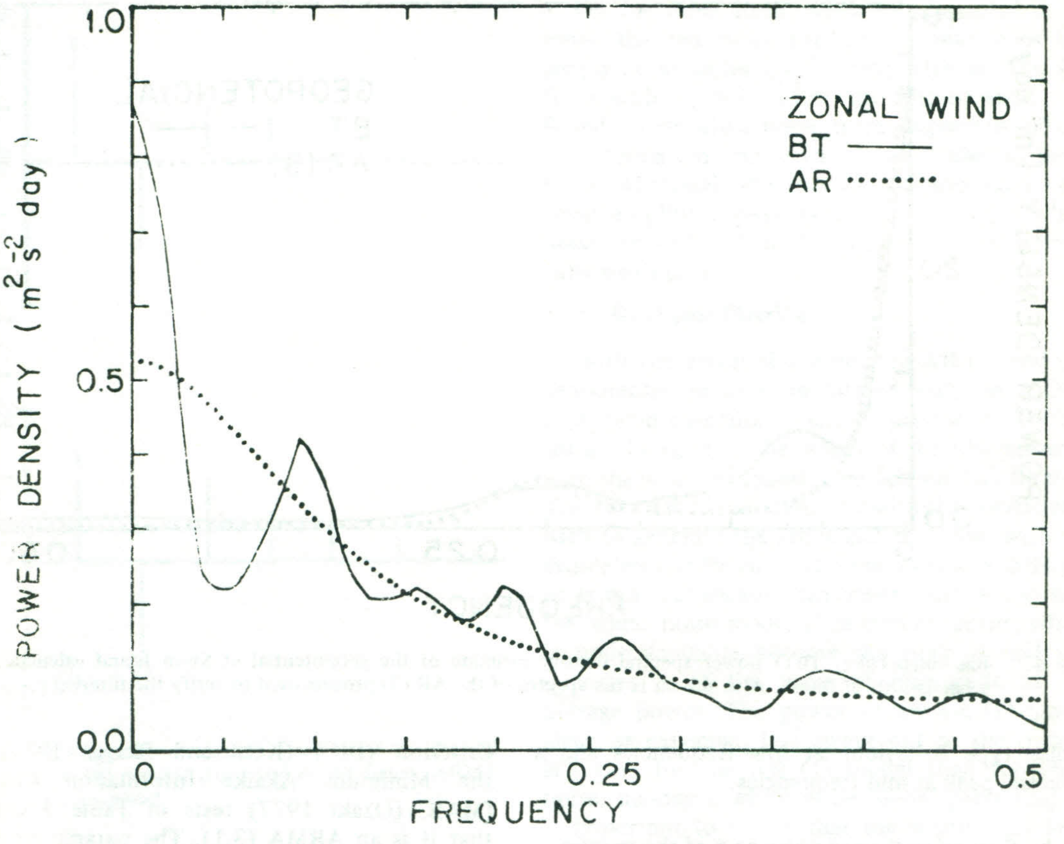

Fig. 2. Blackman-Tukey (BT) and autoregressive (AR) estimates of the power spectral density of the zonal wind component at Swan Island Station. The spectra is plotted versus normalized frequency and the band width of the BT estimate is 0.048.

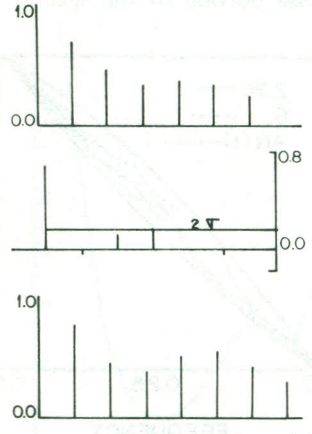

Fig. 3. Autocorrelation (top) and partial autocorrelation (middle) functions of the geopotential at Swan Island Station. Their behaviour suggest an AR (4) model. Also shown is the autocorrelation function (bottom) of the generated AR (3) process.

were generated by processing with suitable AR filters two series of 200 normally distributed numbers with zero mean and unit variance. Only the last 153 numbers were kept. The first series was passed through an order one filter with $a_1 = 0.5$. The Burg estimate of a_1, for the resulting series is 0.489. The second series was passed through an AR filter of order three and coefficients equal to $a_1 = 1.3405$, $a_2 = -1.0866$ and $a_3 = 0.6313$. The autocorrelation function is shown in Fig. 3 and the spectra in Fig. 4. The corresponding transfer function has poles in the complex plane at (0.909, 0^o) and (0.833, $\pm 75^o$). Burg's estimate of the cofficients gives $a_1 = 1.3860$, $a_2 = -1.0540$ and $a_3 = 0.52941$, with poles now at (0.873, 0^o) and (0.778, $\pm 77^o$). Notice that the series used to simulate the geopotential data is an AR(3) process and not an AR(4). The reason for this is that three is the smallest order that is required to generate the red

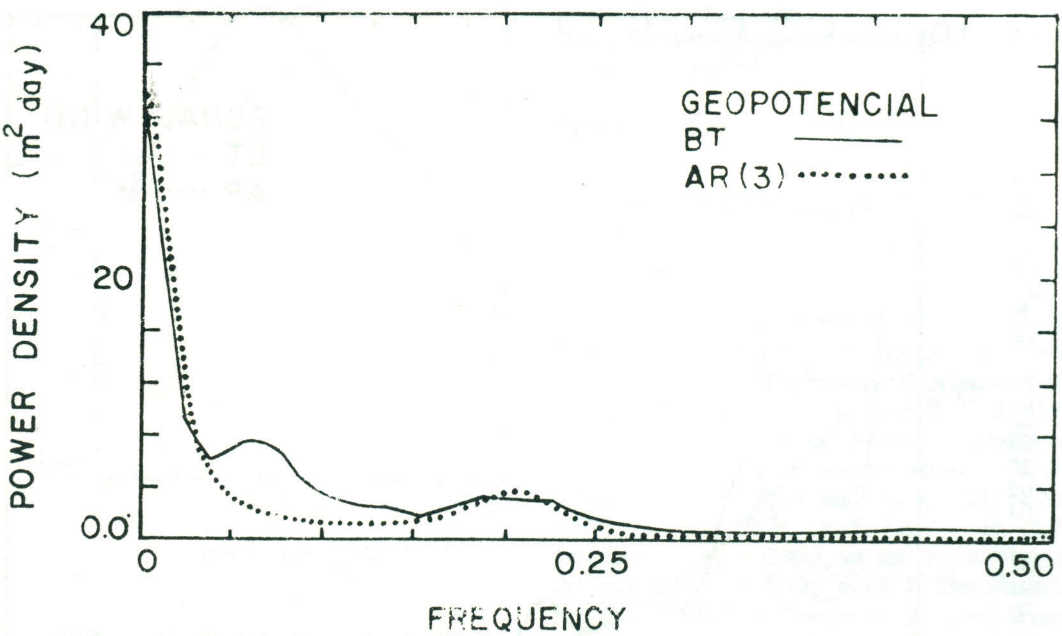

Fig. 4. Blackman-Tukey (BT) power spectral density estimate of the geopotential at Swan Island calculated with a bandwidth of 0.048. Also shown is the spectra of the AR (3) process used to verify the filtering procedures.

noise type behaviour at low frequencies and a spectral peak at mid frequencies.

b. Removing the red noise part of the spectra

As discussed previously, an AR(1) process can be whitened by an MA(1) filter with a moving average coefficient equall to the autoregressive coefficient. The artificially generated AR(1) series was filtered with $b_1 = 0.489$ and the zonal wind component with $b_1 = 0.48287$. The autocorrelation and partial autocorrelation functions for both filtered series followed the expected behaviour of white noise. The white noise test shown in Fig. 5 confirms that both filtered series are indeed white noise.

The artificially generated AR(3) series was similarly passed through an MA(1) filter with $b_1 = 0.873$ to cancel the pole in the positive real axis. The partial autocorrelation (Fig. 6) shows a behaviour which does not correspond to a pure AR process. The exponential type decay after the third partial autocorrelation points to an ARMA (3,1) model, as expected if the pole in the real axis is not exactly cancelled. The GPAC function in Table 1 and the Bayesian Information Criterion (BIC) (Katz and Skaggs 1981) and the Minimum Akaike Information Criterion (MAIC) (Ozaki 1977) tests in Table 2 confirm that it is an ARMA (3,1). The parameters fitted are $b_1 = .690$, $a_1 = 1.500$, $a_2 = {}^{-}1.145$, $a_3 = .568$, and the partial autocorrelation function is shown in Fig. 6. The spectra in Fig. 7 shows nevertheless that red noise portion of the spectra is removed partially.

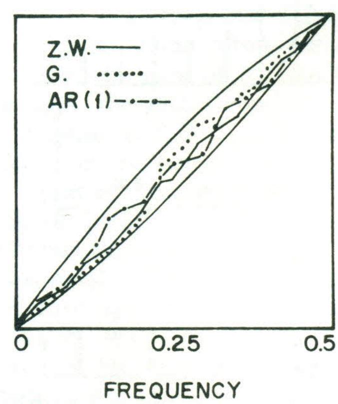

Fig. 5. White noise test of the filtered geopotential and zonal wind component series at Swan Island Station and of the filtered AR (1) process. The significance limits drawn correspond to the 0.05 level.

On the other hand, when we attempted to remove the red noise part of the spectra of the geopotential series by filtering with an MA (1) filter with b_1 = 0.74579, the filtered series was found to be white noise from inspection of the autocorrelation and partial autocorrelation functions. Allthough the spectra still showed a very small amplitude peak near 0.2 1/day, the white noise test in Fig. 5 confirms that it is not statistically significant.

c. Band pass filtering

Both the artificially generated AR (3) and the geopotential series were filtered with the ARMA (4,4) band pass filter described previously. Table 3 shows the ratio of the power of the filtered series with the power of different models similary filtered. The two AR (3) models, one with the coefficients used to generate the series and the other with the estimated coefficients yield results whose difference is not statistically significant. Notice also that the white noise model gives similar results, which is understandable because the peak at mid frequencies is of the order of magnitude of the average power. The power of an AR (1) model gives, as expected, less power out so the q ratio is higher for the AR (1) model than for white noise, making a white noise model preferable. It is important to notice that the significance level is not better tha 5%..

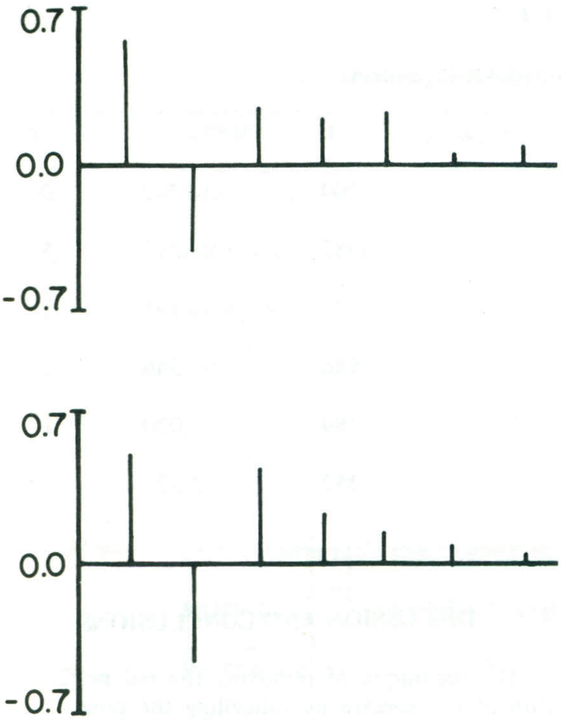

Fig. 6. Partial autocorrelation function of the filtered AR (3) process (top) and of the fitted ARMA (3, 1) model.

Fig. 7. Power spectral density in arbitrary units of the geopotential series filtered by an MA (1) process. The red noise part of the spectrum is removed substantially.

TABLE 1

GPAC Function of filtered AR (3) process

q/p	1	2	3	4	5	6
0	.547	−.427	.257	.204	.242	.035
1	−.001	−.159	.574	−.087	.215	−.512
2	.015	−.162	.604	1.93	.191	−.140
3	−1.97	−8.65	.578	.546	.346	−.256
4	2.46	−1.72	.942	.284	.020	.175
5	.889	−.641	.615	.252	−2.32	.126

TABLE 2

BIC and MAIC of filtered AR (3) process

p	q	BIC	MAICE
3	1	19.828	1.603
3	0	28.486	14.314
3	2	24.683	2.386
2	0	34.047	23.925
2	1	34.271	20.094
2	2	40.262	22.022
4	0	28.346	10.106
4	1	24.905	2.6000
4	2	32.636	6.256

Table 4 shows the q ratio test for the geopotential series. As expected the AR (4) model gives better results than the AR (3) model, but the white noise model cannot be rejected at the 5% significance level. The AR (1) model can be rejected at a significance level much better than 1 %.

4. DISCUSSION AND CONCLUSIONS

The technique of removing the red noise portion of the spectra by cancelling the pole of the transfer function proved to be succesful. In the case of the artificially generated AR (1) process and with the zonal wind component the resulting filtered series were white noise. The results with the artificially generated AR (3) series and the geopotential series need more interpretation. With the AR (3) series, the filtered series was found to be an ARMA (3, 1) process, as is to be expected if no exact cancellation of the pole with the introduced zero occurs. The spectra showed that red noise portion is significantly reduced and thus the peak near 0.2/day is made more prominent. On the other hand, when the red noise part of the geopotential series is removed, the resulting series has no structure and is best modeled by white noise.

The q ratio test of the artificially generated AR (3) series filtered with the ARMA (4, 4) band pass filter shows that the filter's output power is not significantly different for both AR (3) models and the white noise model. The AR (1) model predicts a smaller output power, but the difference is not significant at the 5% level. This result can be explained because at the filter's center frequency the power spectral density of all models is of about the same magnitude. The test would have better resolution at a higher center frequency, where, for example,

TABLE 3

Power ratio significance test of filtered AR (3) process

MODEL	POWER RATIO	SIGNIFICANCE LEVEL
White noise	1.06	.82
AR (3) estimated	1.07	.80
AR (3) known	.900	.72
AR (1)	1.37	.15

TABLE 4

Power ratio significance test of filtered geopotential series

MODEL	POWER RATIO	SIGNIFICANCE LEVEL
White noise	.797	.42
AR (3)	1.36	.15
AR (4)	1.06	.83
AR (1)	1.92	2.2E–04

the power spectral density predicted by the AR (1) model is much lower than the average power spectral density. The same test performed on the filtered geopotential series can only rule out the AR (1) model, but it can not discriminate between the white noise model and the AR (4) model. In the interest of parsimony the white noise model would be prefered, agreeing with the result of removing the red noise component of the spectra.

The tests perfomed give no evidence that the spectral peak near 0.2/day of the geopotential series should be considered real.

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