

UNIVERSIDAD DE COSTA RICA
SISTEMA DE ESTUDIOS DE POSGRADO

AN ONLINE ENGLISH FOR SPECIFIC PURPOSES COURSE FOR MATHEMATICS
PROFESSORS

Trabajo final de investigación aplicada sometido a la consideración de la Comisión del Programa de Posgrado en Enseñanza del Inglés como Lengua Extranjera para optar al grado y título de Maestría Profesional en Enseñanza del Inglés como Lengua Extranjera

ARAYA CEDEÑO, MARILYN
FALLAS BRENES, PAMELA
MONTERO BOLAÑOS, RANDALL

Ciudad Universitaria Rodrigo Facio, Costa Rica

2025

DEDICATORIA

This work is dedicated to God, whose guidance and strength have been the foundation of this journey. It is also dedicated to my husband whose unwavering support, patience, and encouragement have been invaluable. His belief and love have been a constant source of motivation throughout this process.

Marilyn

I dedicate this project to my parents who have always been an example of unwavering hard work. Their sacrifices, strength, and perseverance have shaped the person I am today. To my family and close friends for their endless love and always believing in me. And to my fiancé, for his support, kind words, and encouragement during the toughest times.

Pamela

To my family, for all their support and encouragement throughout my academic life, especially to my grandmother (RIP) and my mother, who have always been the prime example of dedication and perseverance to me.

Randall

AGRADECIMIENTO

First and foremost, gratitude is given to God for His endless wisdom, strength, and blessings throughout this journey. His presence has been a constant source of guidance and resilience, making this achievement possible. To my classmates, thank you for your support and patience. Sharing this experience with them has made every challenge more manageable and every success more meaningful. Finally, to my professors, I deeply appreciate their dedication and guidance. Their passion for teaching and commitment to our growth have been instrumental in shaping this learning journey.

Marilyn

I am forever grateful to my family, friends, and fiancé for always standing by my side, showing their unconditional love and support. I want to thank my colleagues, Marilyn and Randall, for their dedication, collaboration, and kindness. Working alongside such talented, supportive, and responsible people has made all the difference in this journey. I want to express my sincere gratitude to Professor Randolph Zúñiga for his consistent patience and guidance throughout the whole process. I am grateful for the expertise and wisdom he has shared with me, which have inspired me in many ways. Last but not least, thank you to Professors Roxana Chévez, Irene Marín, Alonso Canales, Ana Cristina Alvarado, and Director Xinia Rodríguez for their teachings and valuable feedback

Pamela

Above all, thanks to God for every blessing in my life and to my family for their love and support. Special thanks to my colleagues, Pamela and Marilyn, for their valuable input and constant dedication. Many thanks to professor Randolph Zúñiga for his thoughtful guidance and much-appreciated feedback throughout this process.

Randall

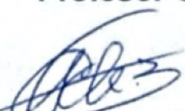
"Este trabajo final de investigación aplicada fue aceptado por la Comisión del Programa de Estudios de Posgrado en Enseñanza del Inglés como Lengua Extranjera de la Universidad de Costa Rica, como requisito parcial para optar al grado y título de Maestría Profesional en la Enseñanza del Inglés como Lengua Extranjera."



Dr. Alonso Canales Víquez
Representante de la Decana Sistema de Estudios de Posgrado



Mag. Randolph Zúñiga Coudin
Profesor Guía



Dra. Ana Vivian Fernández Peraza
Lectora



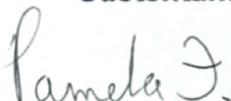
Mag. Roxana Chévez Herra
Lectora



M.A. Xinia Rodríguez Ramírez
Directora Programa de Posgrado en Enseñanza del Inglés como Lengua Extranjera



Marilyn Araya Cedeño
Sustentante



Pamela Fallas Brenes
Sustentante



Randall Montero Bolaños
Sustentante

TABLA DE CONTENIDO

DEDICATORIA.....	ii
AGRADECIMIENTO.....	iii
TABLA DE CONTENIDO.....	v
RESUMEN	viii
ABSTRACT	x
LISTA DE TABLAS.....	xi
LISTA DE FIGURAS	xii
LISTA DE ABREVIATURAS.....	xiii
Chapter I. Needs Analysis	3
Methodology.....	5
Context and Participants.....	5
Procedure.....	6
Research Approach.....	7
Instruments.....	8
Results	10
Interests of Primary Stakeholders.....	10
Group Profile	11
Students' Learning Strategies, Learning Styles, and Attitudes.....	22
Diagnostic Test.....	24
Test Administration Issues.....	24
Diagnostic Test Design.....	25
Diagnostic Test Results.....	29
Chapter II. Syllabus Design	40
Course Logo.....	40
Course Description	40
Statement of Goals and Objectives	41
Unit 1: Delivering Arithmetic, Algebra, and Geometry Input	41
Unit 2: Delivering Arithmetic, Algebra, and Geometry Lessons	42
Unit 3: Delivering Basic Geometry and Trigonometry Lessons	42
Methodology.....	43
Approach.....	43
Classroom Dynamics.....	44
Tasks and Techniques.....	44
Role of the Learner.....	45

Role of the Instructors.....	45
Assessment.....	46
Assessment Tasks	47
Contents.....	51
Unit 1: Delivering Arithmetic, Algebra, and Geometry Input	51
Unit 2: Delivering Arithmetic, Algebra, and Geometry Lessons	54
Unit 3: Delivering Basic Geometry and Trigonometry Lessons	58
Conclusion	61
Chapter III. Practicum Research.....	62
Literature Review	63
Key Concepts	63
Main Challenges.....	68
Needs Analysis and Course Design.....	69
Task-Based Language Teaching (TBLT) and Scaffolding.....	70
Technology Integration	73
Differentiated Instruction.....	74
Formative and Summative Assessment.....	75
Balancing Synchronous and Asynchronous Instruction.....	75
Cooperative Learning	76
Methodology.....	78
Research Approach.....	78
Context.....	79
Instruments.....	80
Procedures	84
Results	86
Formative Assessments	87
TBLT and Balancing Synchronous and Asynchronous Instruction	90
Technology Integration	94
TBLT Scaffolding Instruction Principle	95
Cooperative Learning	97
Differentiated Instruction.....	104
Summative Assessment	108
Discussion.....	115
Formative Assessment	115
TBLT and Balancing Synchronous and Asynchronous Instruction	116
Technology Integration	117
TBLT Scaffolding Instruction Principle	118

Cooperative Learning	119
Differentiated Instruction.....	121
Summative Assessment	121
Conclusions.....	122
Limitations	124
Recommendations	125
References.....	128
Appendix A.....	132
Appendix B.....	134
Appendix C	138
Appendix D	145
Appendix E.....	149
Appendix F.....	151
Appendix H	156

RESUMEN

Este portafolio documenta el diseño, implementación y evaluación de un curso en línea de Inglés con Fines Específicos (ESP, por sus siglas en inglés) desarrollado para docentes de matemáticas en la Sede de Occidente de la Universidad de Costa Rica durante el segundo semestre de 2023. La investigación examina específicamente el impacto de las estrategias de instrucción diferenciada en la realización de tareas en un aula virtual de ESP con niveles mixtos de competencia lingüística. Mediante una metodología cualitativa, el estudio recopiló datos de múltiples fuentes, incluyendo encuestas a estudiantes, entrevistas a supervisores, observaciones de los instructores y evaluaciones orales preliminares y finales. El diseño del curso integró la Enseñanza Basada en Tareas (TBLT, por sus siglas en inglés) con técnicas de instrucción diferenciada para adaptarse a los diversos niveles de competencia. Las estrategias clave incluyeron agrupamientos flexibles, materiales de apoyo escalonado y una metodología equilibrada entre instrucción sincrónica y asincrónica. Los resultados indican que los enfoques diferenciados impactaron positivamente en la realización de tareas y el desempeño general de los estudiantes, mostrando tanto los aprendices de nivel básico como intermedio una mejora notable en el uso de vocabulario matemático y habilidades de producción oral. Los estudiantes expresaron preferencia por grupos homogéneos, aunque también reconocieron los beneficios de la colaboración entre diferentes niveles de competencia. Los hallazgos sugieren que la implementación sistemática de instrucción diferenciada mediante materiales adaptados y agrupamiento estratégico puede apoyar efectivamente a los estudiantes de ESP con diferentes niveles de competencia en entornos virtuales. Esta investigación aporta perspectivas prácticas para profesionales de ESP que trabajan con grupos de habilidades mixtas en entornos virtuales.

Palabras clave: Inglés con Fines Específicos (ESP), Instrucción diferenciada, Enseñanza a niveles mixtos de competencia, Enseñanza virtual de idiomas, Educación matemática, Enseñanza de idiomas basada en tareas, Diseño de cursos en línea o virtuales

ABSTRACT

This portfolio documents the design, implementation, and evaluation of an online English for Specific Purposes (ESP) course developed for mathematics faculty at the Western Campus of the University of Costa Rica during the second semester of 2023. The research specifically examines the impact of differentiated instructional strategies on task completion in a mixed-proficiency virtual ESP classroom. Through qualitative methodology, the study collected data from multiple sources including student surveys, supervisor interviews, instructor observations, and pre/post speaking assessments. The course design integrated Task-Based Language Teaching with differentiated instruction techniques to accommodate varying proficiency levels. Key strategies included flexible grouping arrangements, scaffolded materials, and balanced synchronous/asynchronous delivery. Results indicate that differentiated approaches positively impacted students' task completion and overall performance, with both basic and intermediate-level learners showing marked improvement in mathematical vocabulary usage and oral production skills. Students expressed preference for homogeneous grouping while also acknowledging benefits of mixed-ability collaboration. The findings suggest that systematically implementing differentiated instruction through tailored materials and strategic grouping can indeed support ESP learners across proficiency levels in virtual environments. This research contributes practical insights for ESP practitioners working with mixed-ability groups in online settings.

Keywords: English for Specific Purposes (ESP), differentiated instruction, mixed-proficiency instruction, virtual language teaching, mathematics education, task-based language teaching, online course design

LISTA DE TABLAS

Tabla 1. Population's English-learning Needs According to Stakeholders.....	11
Tabla 2. Listening Section, Task 1 Result.....	29
Tabla 3. Listening Section, Task 2 Results.....	31
Tabla 4. Reading Section, Task 1 Results.....	32
Tabla 5. Reading Section, Task 2 Results.....	34
Table 6. Summary of Performance Levels in the Reading and Listening Tasks.....	36
Table 7. Student's Performance in the First Two Speaking Tests.....	37
Table 8. Course Evaluation.....	47
Table 9. Main Task Description per Week (practice tasks).....	87
Table 10. Students' Perceptions of their Preparedness, Participation, and Engagement in the Course.....	89
Table 11. Students' Opinions about the Course	91
Table 12. Students' Opinions about their Most Enjoyable Part of the Course	93
Table 13. Students' Opinions about Course Improvements	98
Table 14. Students' Feelings when Working with Classmates with the Same or Similar English Proficiency Level	100
Table 15. Students' Feelings when Working with Classmates with a different English Proficiency Level	101
Table 16. Preference between Working with Classmates who Have a Similar or Different English Proficiency	102
Table 17. Students' Opinions on Differentiated Worksheets according to their Specific English Language Proficiency	104
Table 18. Comparison of Students' Performance on Diagnostic Test and Final Speaking Test	109

LISTA DE FIGURAS

Figure 1. Learn Mathematics Vocabulary	12
Figure 2. Pronounce Mathematics Vocabulary Correctly	12
Figure 3. Participate in Conversations Related to Mathematics	13
Figure 4. Write Texts and Essays about Topics Related to Mathematics	14
Figure 5. Easily Understand Mathematical Procedures Explained by English Speakers	14
Figure 6. Explain Different Mathematical Procedures Fluently	15
Figure 7. Give Instructions Related to Mathematical Procedures Fluently	15
Figure 8. Teaching-Learning Preferences	16
Figure 9. Course Expectations: Skills Participants Would Like to Address	17
Figure 10. Mathematics Content: Participants' Preferences for Primary Course Focus	18
Figure 11. Participants' Priorities about Mathematics Content Areas	18
Figure 12. Participants' Self-reported English-Speaking Proficiency	20
Figure 13. Participants' Self-reported English Writing Proficiency	21
Figure 14. Participants' Self-reported English Listening Proficiency	21
Figure 15. Participants' Self-reported English Reading Proficiency	22
Figure 16. Enjoy Taking an English Course	23
Figure 17. Learn English as an Obligation and Not as a Personal Interest	23
Figure 18. Enjoy Being in Contact with the Language Outside of Class	24
Figure 19. Like the Idea of Integrating English into Math Lessons	24

LISTA DE ABREVIATURAS

ESP: English for Specific Purposes

ESL: English as a Second Language

TBLT: Task-Based Language Teaching

UCR: University of Costa Rica

POC: Point of Contact

CEFR: The Common European Framework of Reference for Languages

Abi: Alliance for Bilingualism

T = Teacher

ATs = Assistant Teachers

Ss = Students

BOR = Breakout Room

Min = Minutes

An Online English for Specific Purposes Course for Mathematics Professors

This portfolio serves as a comprehensive compilation of work conducted throughout the course, reflecting the design, implementation, and evaluation of an online English for Specific Purposes (ESP) course tailored to faculty members of the Mathematics Department at the Western Campus of the University of Costa Rica during the second semester of 2023.

As part of the course design process, the ESP course was structured into three units, each focusing on specific language skills and mathematical contexts to support targeted learning objectives. The course-design team developed instructional materials, assessments, and strategies to address the diverse proficiency levels of participants. The practicum phase involved the implementation of the course, allowing student-teachers to apply differentiated instructional strategies in a virtual classroom setting. Through this experience, they engaged in instructional delivery, classroom management, and adaptation of materials based on student needs. The practicum research component of this qualitative study explores the impact of differentiated instructional strategies on students' task completion in a mixed-proficiency virtual ESP class. To gather insights, we employed data collection methods such as surveys, observations, and interviews, analyzing student engagement and the effectiveness of instructional approaches.

The document is organized into several chapters, each focusing on a crucial component of the overall study. Chapter 1: Needs Analysis examines the initial assessment of mathematics professors' language requirements, instructional gaps, and contextual factors influencing curriculum design. This section provides insights into the foundational research that informed the development of the syllabus. Chapter 2: Syllabus Design presents the curriculum framework developed based on the needs analysis. It includes the rationale for content selection, instructional strategies, and assessment methods used to ensure effective language acquisition within a mathematical context.

Chapter 3: Practicum Research documents the practical application of the designed syllabus, including its implementation in an online learning environment. This section analyzes the effectiveness of instructional strategies, challenges encountered, and adjustments made to enhance student engagement and learning outcomes.

The portfolio concludes with a Conclusions section, summarizing key findings and reflections on the overall study. Recommendations are then provided to suggest improvements for future curriculum development and instructional practices. Lastly, the References section lists all sources cited, and the Appendices include supplementary materials that support the research and implementation process. This portfolio aims to serve as a resource for educators and researchers interested in developing online ESP courses, specifically for mathematics professors, showcasing a structured approach to designing and evaluating specialized language instruction.

Chapter I. Needs Analysis

Mathematics is often viewed as universal and free of cultural influences.

Mathematics instruction and mathematical symbols are common across cultures, so, typically, language learners are already familiar with mathematics by the time they learn a foreign language. However, the language used to communicate mathematical concepts, ideas, problems, and procedures is far from being a universal one. Within mathematics education research, various authors have even described mathematics as a language in and of itself and emphasized the importance of developing the students' ability to communicate mathematically (Jarrett, 2009; Jourdain & Sharma, 2016; Ernst-Slavit & Slavits, 2023). In addition to everyday language, the language of mathematics consists of technical terms and symbols, as well as grammatical conventions and structures unique to mathematical discourse. These particular features of the language of mathematics can make it difficult for English language learners to communicate in the language of mathematics using a foreign language that they have not yet mastered (Jourdain & Sharma, 2016).

One factor that can pose a major challenge for foreign language learners is that mathematical operations can be expressed in different ways. For example, addition can be signaled with words like add, plus, combine, sum, and increased by. What is more, some mathematical symbols and conventions differ from English-speaking countries to Spanish-speaking countries. For example, while a comma is used to separate whole numbers from decimal parts in Costa Rica, a point is used instead as the decimal separator symbol in the United States. Also, while large numbers, like thousands and millions, are separated using periods in Costa Rica, they are separated using commas in the United States.

Another factor that can pose a significant challenge for foreign language learners is that the way mathematical concepts are expressed in words often differs in its order from the way the concepts are expressed in symbols (Ernst-Slavit & Slavits, 2023). As a result,

foreign language learners may attempt to read and write mathematical sentences in the same way that they read and write standard narrative text. That is, they may try to translate word-for-word between a mathematical concept expressed in words and the concept expressed in symbols, but a linear, one-to-one translation is often incorrect. For example, the authors illustrate this difference with an algebraic phrase where the number 'a' is five less than the number 'b', the student may mistakenly restate it as $a=5-b$, when it should be $a=b-5$. Additionally, it would take several paragraphs to solve a problem or equation using standard narrative text. Therefore, it is fundamental to expose second language learners to target input at an early stage so that they can get acquainted with how mathematical concepts are verbally expressed in English.

In addition to how mathematical concepts are verbally expressed in English, other features specific to the spoken language of mathematics may present difficulties for foreign language learners. For example, language learners may have difficulties using discourse connectors properly when explaining mathematical solution procedures. In mathematics, the way discourse connectors signal similarity and contradiction, cause and effect, reason and result, and chronological sequence may vary from the way discourse connectors signal logical relationships between parts of a text in everyday language. Similarly, the particular use of prepositions and comparative structures like greater than or less than in mathematical discourse may pose additional challenges for students trying to learn English as a foreign language and mathematics at the same time (Ernst-Slavit & Slavit, 2023). Thus, it is essential to expose second language learners to mathematical solution procedures also at an early phase to raise awareness of how discourse connectors and comparative structures are specifically used in mathematical discourse.

Although mathematics has traditionally been regarded as an area with minimal language demands, mathematics and language are intricately connected, as language facilitates mathematical thinking and problem solving (Jourdain & Sharma, 2016).

Mathematics instruction in a foreign language should thus be based on inquiry and problem-solving tasks, which are permeated with specific speaking and listening skills, such as questioning, describing, explaining, hypothesizing, debating, clarifying, elaborating, and verifying and sharing results. As can be seen, while the language demands can be substantial for students, the potential to learn essential communicative skills along with mathematics content is high.

On August 13, 2018, former President of the Republic of Costa Rica, Carlos Alvarado, introduced the Alliance for Bilingualism (ABi) as part of a national strategy that seeks to significantly increase the coverage of English teaching in Costa Rica (Cinde, 2018). With country-wide initiative in mind, the Western Branch of the University of Costa Rica is designing a new bachelor's degree program in bilingual mathematics instruction, the first of its kind in the country. This curriculum-design project, still at an initial planning phase, aims to first prepare their Mathematics Department Faculty to become bilingual educators so that undergraduate students enrolled in this new major can take mathematics courses taught in English by the mathematics professors themselves. As part of this initiative, the course-design team worked very closely with faculty members to design and teach an English for Specific Purposes (ESP) course to mathematics professors. To this end, they have followed the methodology outlined below to gather data for the preliminary needs analysis.

Methodology

Context and Participants

As stated above, the context of the study is the design of an ESP course for the Mathematics Department Faculty at the Western Campus of the University of Costa Rica, which is be part of a curriculum-design project that aims to prepare math students to be bilingual educators as part of a new bachelor's degree program in bilingual mathematics teaching. This project stems from a curriculum evaluation process of their bachelor's

degree in the teaching of mathematics. Among the areas for improvement this committee identified in their current program, the need to prepare bilingual math teachers was the most evident and ambitious one.

This curriculum-design project is still at an initial stage, however. The Mathematics Department Faculty have not yet designed the specific program, but their plan is for math students to take not only language courses but also math courses taught in English by the math professors themselves. Therefore, the learner population of the ESP course includes 16 mathematics professors of bachelor's degree program in math education, with the exception of a Physics professor from the Natural Science Department. Furthermore, they all specialized in particular subfields, including theoretical mathematics, applied mathematicians, and mathematics education.

Despite the different areas of specialization, the learner population can be considered somewhat homogeneous. This is because most participants are in the same age range and share just about the same experience learning the target language. The participants affirmed that they did not receive an early bilingual education and started to have their first contact with English in high school. The applied and pure mathematicians may have stronger English reading and writing skills because most of their research and publications are in English, but they still lack communicative skills and verbal confidence and fluency in the target language.

Procedure

The first step to start collecting data was to schedule via email the initial semi-structured interview with the three stakeholders from the Mathematics Department Faculty responsible for designing the program in bilingual mathematics teaching. The Director of the Mathematics Department, one of these three stakeholders, acted as the point of contact (POC) for the course-design team. The role of the POC was to help coordinate with the faculty participants the administration of the different needs analysis instruments.

For instance, a week after the semi-structured interview, the POC communicated via WhatsApp with the participants to agree on a date and time for the administration of the needs analysis survey. Then, two weeks after the administration of the survey, the POC also coordinated with the participants and the course-design team the administration of the subsequent diagnostic test. The course-design team communicated with the POC via the institutional email account and administered the needs analysis instruments via the teleconferencing platform Zoom.

Research Approach

ESP is a learner-centered approach to English learning. For that reason, the course-design team conducted a needs analysis to achieve a thorough understanding of the learner population. The data collected from the needs analysis instruments enabled the course-design team to define relevant goals and objectives that correspond to participants' field-related English learning needs. These data in turn helped the course-design team to ensure that course syllabus and instruction are designed and aligned accordingly.

To collect the data for the needs analysis, the course-design team followed a mixed methods research approach in which they collected quantitative and qualitative data simultaneously and analyzed them separately, drawing on the strengths of both approaches (Creswell, 2015). This mixed approach allowed the course-design team to accurately identify the needs, lacks, and wants of the learner population through a careful analysis of the data obtained from an initial semi-structured interview, a follow-up survey, and a diagnostic test that the course-design team themselves carefully designed and validated. The design and validation of these needs-analysis instruments began with consulting the available literature, including the course reader, to obtain theoretical support for what constitutes a needs analysis and the respective instrument design and to determine the types of instruments and specific items to be included. These quantitative

and qualitative instruments provided the course-design team with valuable insights into the needs, lacks, and interests of the learner population, which will be detailed later in this report.

Instruments

Semi-structured Interview. The first needs analysis instrument, the semi-structured interview, included a set of preplanned, open-ended questions, which was validated among course-design team members (see Appendix A). They asked the interview questions among themselves to ensure that every one of them was relevant, succinct, and specific enough. Thanks to this semi-structured interview, which lasted about an hour, the course-design team obtained general information about the current mathematics undergraduate program and the learner population, including their experiences and needs regarding the use of the target language and their overall expectations about the ESP course. The data collected through this qualitative method guided the design of the subsequent survey, as well as the creation of an initial profile of the learner population.

Survey. The second needs analysis instrument, the survey, featured a series of questions that aimed to collect data from respondents about their attitudes, opinions, and experiences learning the target language (see Appendix B). This instrument was first revised by the course-design professor, who provided detailed feedback on both content and format, and then validated by a member of the Mathematics Department Faculty. This faculty member, who did not take part in the ESP course, provided a list of specific recommendations regarding the format and content of the survey, which the course-design team incorporated into the final version of the instrument. Thanks to this validation step, the course-design team was able to not only improve the format of the instrument but also the content by adding a relevant section related to the level and area of mathematics that the respondents consider a top priority for the future ESP course. After this step, the course-design team recreated the survey in Google Forms for easier test administration

and data collection. To administer this second instrument, the course-design met with the learner population to go over general and specific test instructions and answer questions as needed so that test-takers would feel more at ease, following the principle of reliability (Brown, 2010). Completing the survey took the participants about half an hour. The data gathered from this quantitative instrument, once downloaded as an Excel file from Google Forms, was coded by using descriptive labels to identify trends and communalities amongst the respondents.

Diagnostic Test. The third and last instrument, the diagnostic test, consisted of a listening, a speaking, and a reading section (see Appendix C). Writing was intentionally left out because the results of the two previous instruments showed that listening and speaking skills constitute the top priority for the respondents, particularly for the ESP course. Developing reading skills is not a priority for the ESP course either, but the course-design team still included two reading tasks in the diagnostic test because they would use texts as a source of input in the future ESP course. Unfortunately, the course-design team was not able to validate the diagnostic test with any mathematics faculty members due to time and resource constraints. Nevertheless, the test was first validated through peer assessment by another course-design team following a checklist based on Brown's Principles of Language Assessment (Brown, 2010).

This peer-assessment validation step allowed the course-design team to ensure that the diagnostic test meets the assessment principles of validity, authenticity, and reliability. For instance, the course-designed team was able to confirm that the tasks measure relevant variables and sample performance that reflects the intended type of ESP course. In other words, they were able to confirm that the diagnostic test follows the principle of test validity, which Brown considers the most complex but most important principle (Brown, 2010). The peer-feedback received also helped the course-design team to confirm test authenticity, more specifically that the tasks reflect real-world situations and

contexts like those the students are faced with in an ESP context. The peer-assessment checklist also included points related to construct, face, and content validity, which helped the course-design team to corroborate that the tasks are an accurate sample of the skills the test seeks to assess, constitute a realistic learning sample in relation to the future ESP course, and assess language ability on the basis of the skills to be taught. After the peer-assessment, the course-design professor revised the instrument and provided feedback related to test reliability and practicality (Brown, 2010). For example, he provided several suggestions on how to make the task directions clearer for test-takers and how to be more mindful of the time and effort involved not only for test-takers but also for test-scorers.

Results

Interests of Primary Stakeholders

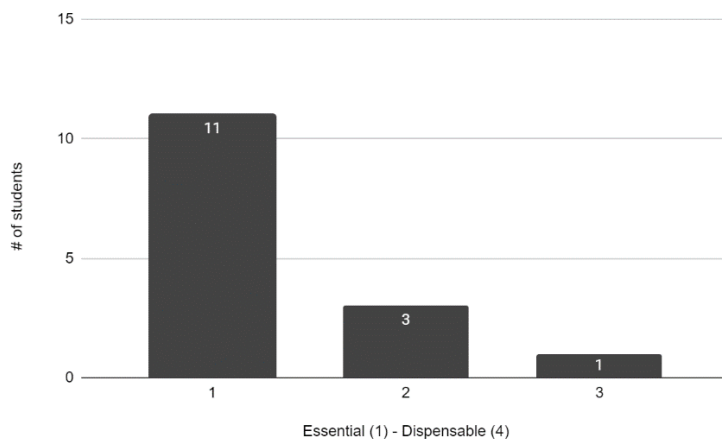
In the initial semi-structured interview, the stakeholders shared with the course-design team the population's needs for the use of English. As previously mentioned, the participants are part of a curriculum-design project that aims to prepare mathematics students to become bilingual educators as part of a new bachelor's degree program in bilingual mathematics teaching at the Western Branch of the University of Costa Rica. Therefore, the mathematics faculty have the need to learn English to teach a set of core mathematics courses in English. Table 1 shows the answers they provided about their specific English language needs. As can be seen, the stakeholders reported that speaking should be the focus of the course, considering that their main task is to explain mathematical procedures orally. They did not consider writing and reading as a priority, at least for the future ESP course. With potential time constraints in mind, the stakeholders expressed the need to prioritize the learning of basic mathematics vocabulary and pronunciation rather than focusing on reading or writing.

Table 1*Population's English-learning Needs According to Stakeholders*

Skill	Needs and Areas of Emphasis Reported by the Mathematics Department Faculty
Speaking	<ul style="list-style-type: none"> -Learning basic mathematics vocabulary. -Pronouncing mathematics vocabulary accurately. -Using mathematics vocabulary in context correctly. -Fully explaining a mathematical procedure orally.
Writing	<ul style="list-style-type: none"> -Writing is not a priority. Some participants have good writing skills because they have written their dissertations and research papers in English.
Reading	<ul style="list-style-type: none"> -Reading is not a priority. Some participants have good reading skills since much of what they read is in English. Therefore, due to time constraints, vocabulary and pronunciation skills are out of scope.

Group Profile

Participants' Needs. The following figures show several academic and future job-related tasks that the participants may need to perform in English. In the needs analysis survey, 15 out of the 16 participants ranked each task on a scale from 1 to 4, where 1 means "essential" and 4 means "dispensable" (one participant was on vacation when the course-design team administered this instrument). Figure 1 shows most participants considered learning mathematics vocabulary as an essential skill.

Figure 1*Learn Mathematics Vocabulary*

As shown in Figure 1, most participants considered pronouncing mathematics vocabulary as a priority. The task of fluently participating in conversations related to mathematics did not receive as much relevance compared to the tasks in Figure 6 and Figure 7. However, most participants still considered this task as essential. The other participants selected option 2 as shown in Figure 2.

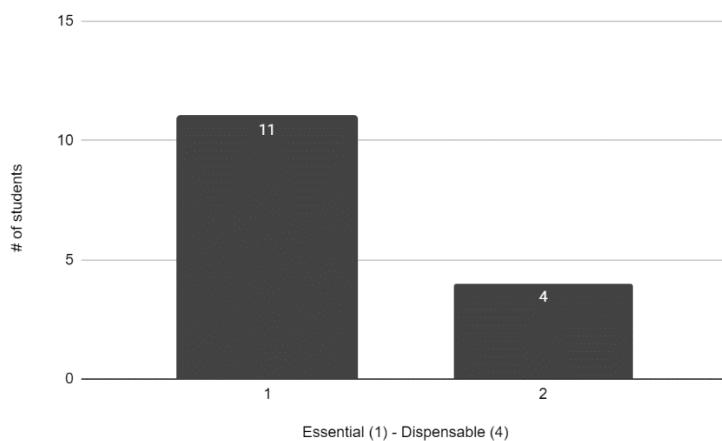
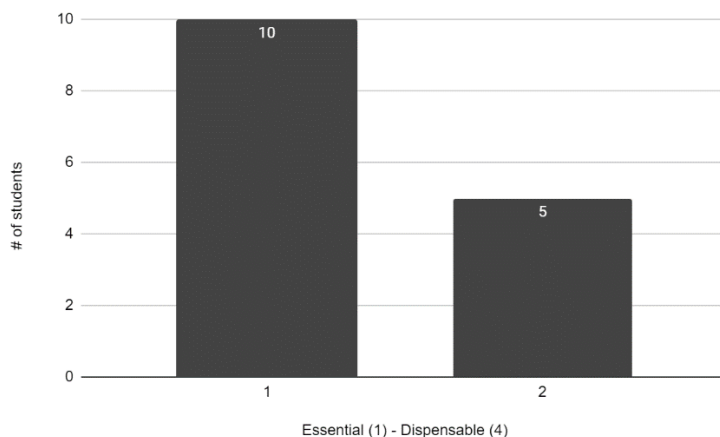
Figure 2*Pronounce Mathematics Vocabulary Correctly*

Figure 3

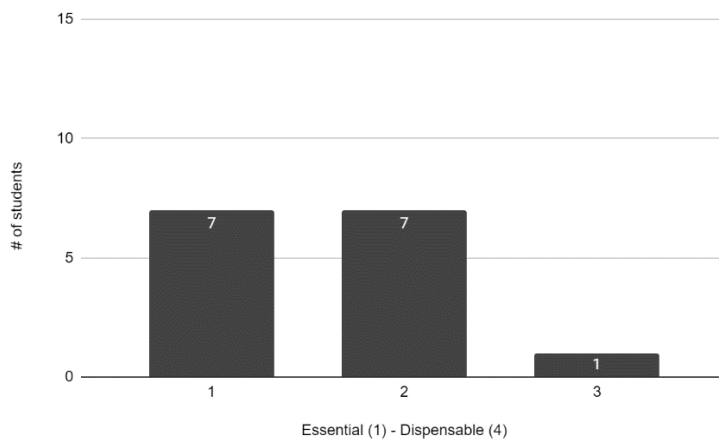
Participate in Conversations Related to Mathematics



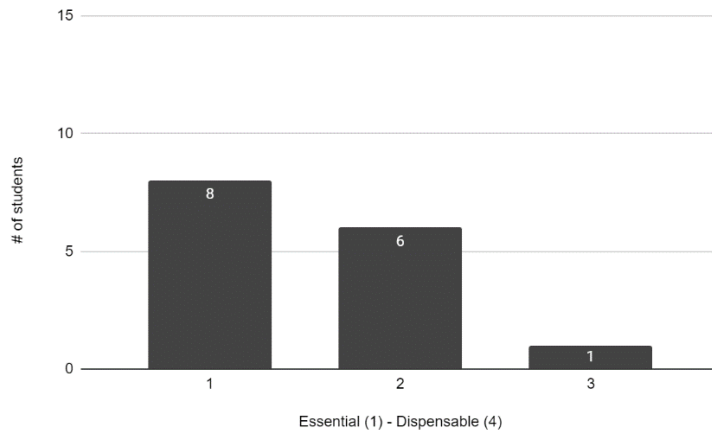
The least essential tasks are shown in Figure 4 and Figure 5. Figure 4 shows how less than half of the participants considered writing texts and essays about topics related to mathematics as an essential task, while the other half considered writing skills unimportant or irrelevant for the future ESP course. These data confirm the results found in the initial semi-structured interview, where the primary stakeholders affirmed that writing and reading skills were not a priority. Similarly, Figure 5 indicates less interest in the task of understanding mathematical procedures explained by English speakers than the course-design team expected, where half of the participants considered it essential.

Figure 4

Write Texts and Essays about Topics Related to Mathematics

**Figure 5**

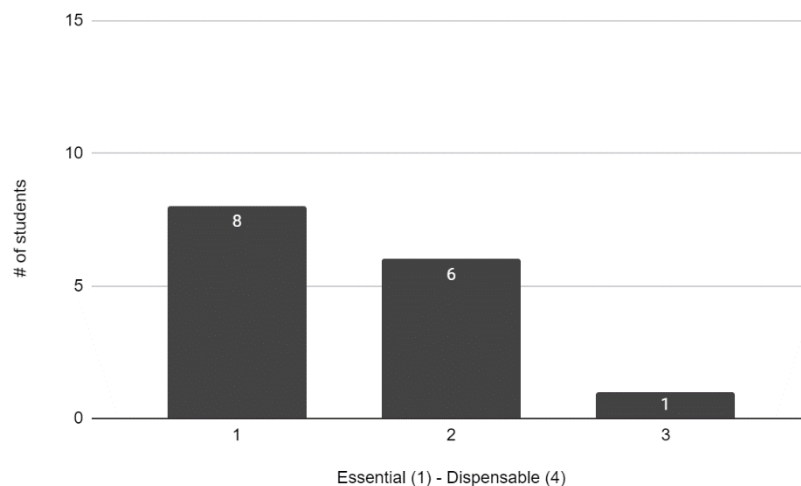
Easily Understand Mathematical Procedures Explained by English Speakers



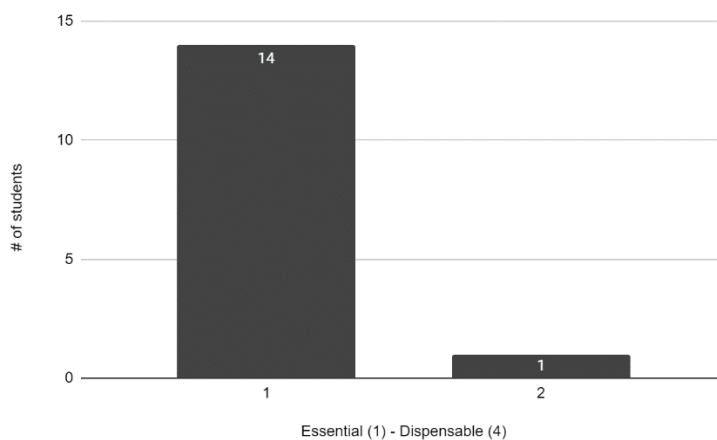
Figures 6 and Figure 7 show the two tasks that participants considered the most essential, namely explaining mathematical procedures and giving instructions related to mathematical procedures fluently. Nearly all participants agreed that these tasks are essential.

Figure 6

Explain Different Mathematical Procedures Fluently

**Figure 7**

Give Instructions Related to Mathematical Procedures Fluently

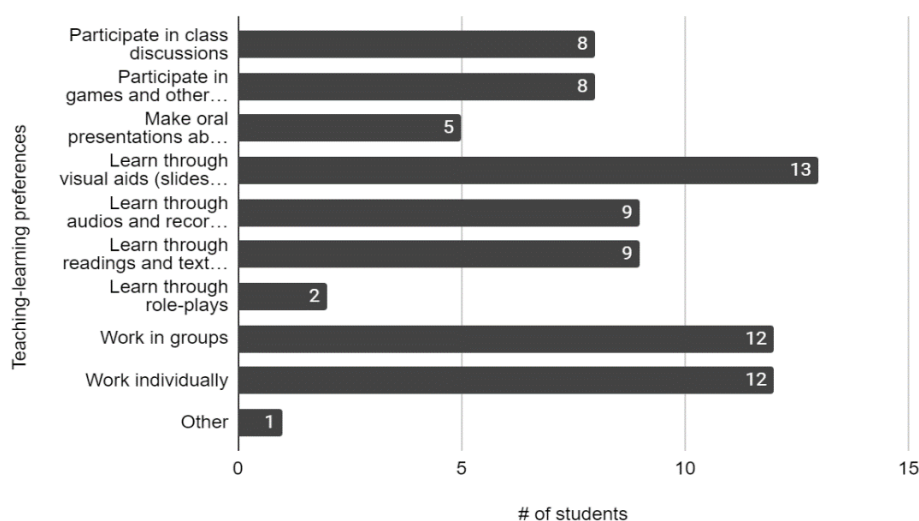


Participants' Wants. Figure 8 indicates the population's teaching-learning preferences. As shown, learning through visual aids, including slideshows, pictures, and videos emerged as the participants' most frequent preference (13), closely followed by both working in groups and working individually (12). Learning through audios and recordings and learning through readings and text analysis were tied in third place (9). The

least frequent preferences reported by the participants were learning through oral presentations (5) and role-plays (2). One participant left a comment in this section of the survey: “No es que no disfrute las actividades, mi nivel de inglés es tan bajo que me da pena” (it is not that I do not enjoy these activities, but my English proficiency is so low that it is embarrassing). In the initial interview, the math faculty committee expressed the same concern about lack of confidence when using the language orally. This probably explains why most participants chose role-plays and oral presentations as their least desired learning activities.

Figure 8

Teaching-Learning Preferences

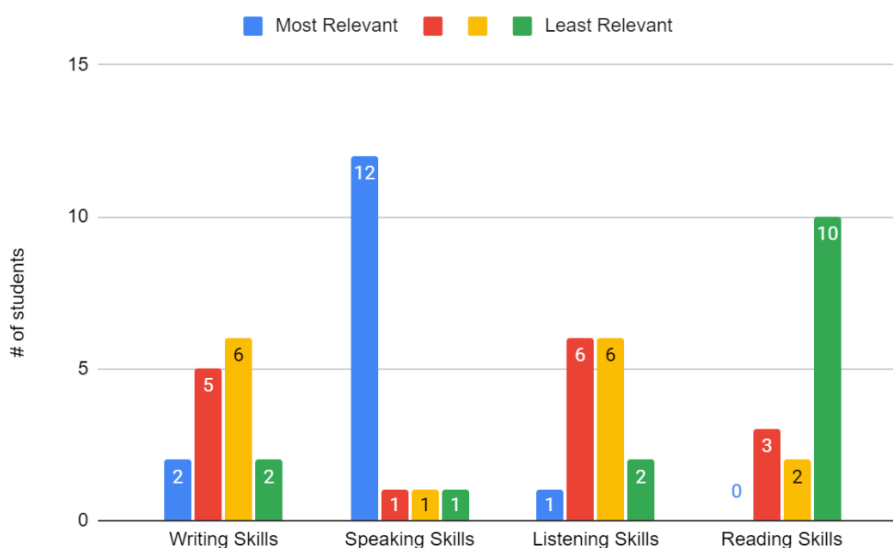


In relation to the skills participants would like to address during the course, Figure 9 confirms the data from the initial interview that speaking is the main macro skill participants want to develop in the future ESP course. It also confirms reading skills as the least desired and writing and listening skills somewhere in between. Data from both needs-analysis instruments suggest that many of the participants might feel intimidated or uncomfortable when performing speaking tasks, but it is the skill they also consider the

most relevant. Therefore, even though they do not consider listening and reading as relevant, they may need to receive enough input through listening and reading to feel successful and build confidence when performing speaking tasks.

Figure 9

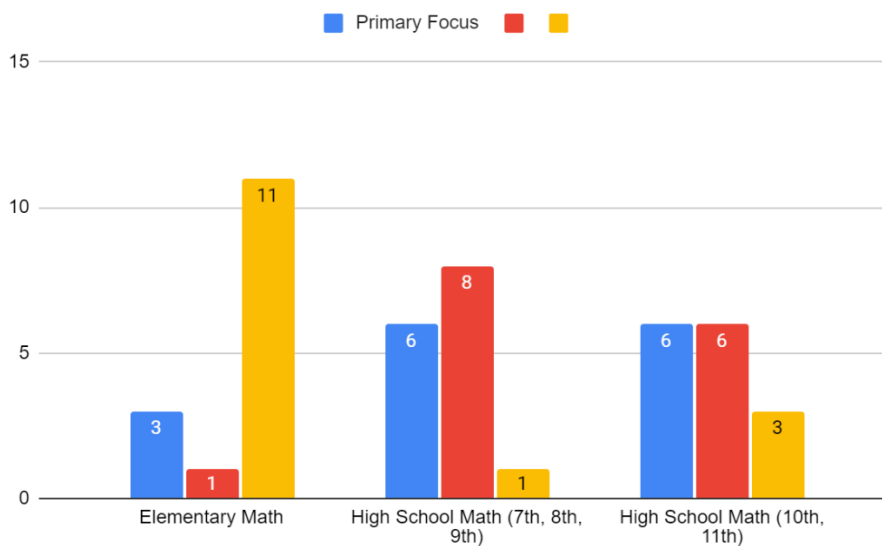
Course Expectations: Skills Participants Would Like to Address



In regard to the primary focus of the mathematics content for the future ESP course, Figure 10 shows that elementary mathematics is the participants' least essential focus, while 7th, 8th, and 9th-grade high school mathematics seems to be the primary focus, followed by 10th and 11th-grade high school math. Interestingly, when asked about other focus areas for the course not included in this section, 9 participants stated their interest in focusing the course on college-level mathematics. These answers did not match the needs that the stakeholders mentioned in the semi-structured interview. On the contrary, the stakeholders seemed realistic about the skills they currently possess, well aware of time constraints, and even suggested the need to start with very basic mathematics terminology. Thus, it is surprising to notice such a large interest in focusing the course on more advanced mathematical content.

Figure 10

Mathematics Content: Participants' Preferences for Primary Focus of the ESP Course

**Figure 11**

Participants' Priorities about Mathematics Content Areas

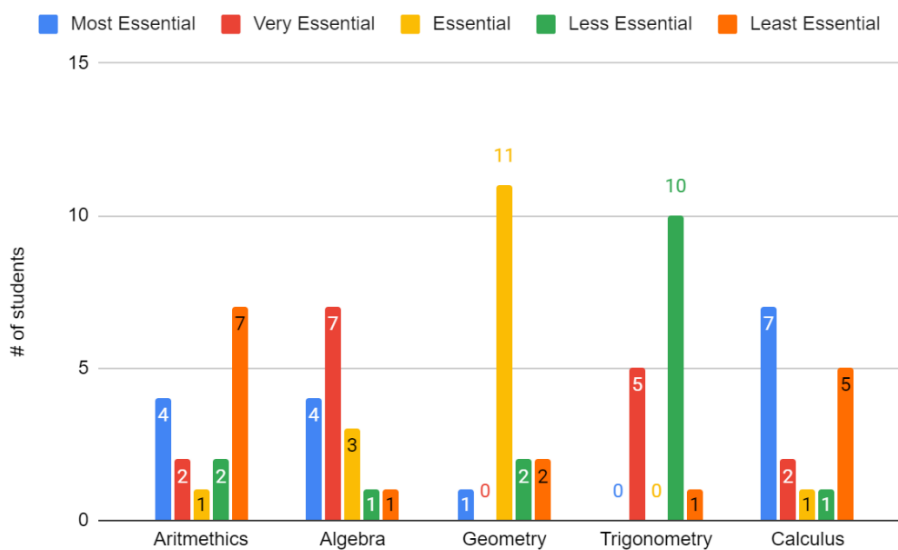


Figure 11 shows the participants' priorities regarding mathematics content areas for the future ESP course. As can be seen, calculus is considered as the most essential

mathematics content area (7 participants). However, combining the answers for top priority and next priority in order of importance, algebra has 11, calculus 9, and arithmetic is third with 6 answers. Additionally, arithmetic also had the most answers for last priority, showing that priorities concerning arithmetic are very divided. With regard to geometry, this area was considered as the third priority by the largest group (11), suggesting that it may also be relevant for the course. Altogether, these results suggest a discrepancy between the semi-structured interview results and the survey results. The results from the initial interview suggested basic mathematics content such as arithmetic as an ideal starting point. In contrast, the results from the survey indicate starting from higher levels of math, including algebra and calculus.

Participants' Lacks. In the semi-structured interview, as stated above, the stakeholders expressed, on behalf of the entire faculty, their feelings about their lack of confidence when using the target language orally. Likewise, in the survey, one participant shared the same concern. This means that for some participants, the affective filter may be high, potentially affecting their English learning. Figure 12 shows students' own perspective regarding their English-speaking proficiency. The vast majority of the participants indicated having deficient English-speaking skills; two indicated acceptable, and only one regarded them as very good. These data suggest that of the all four macro skills, speaking is the English skill participants believe they lack the most.

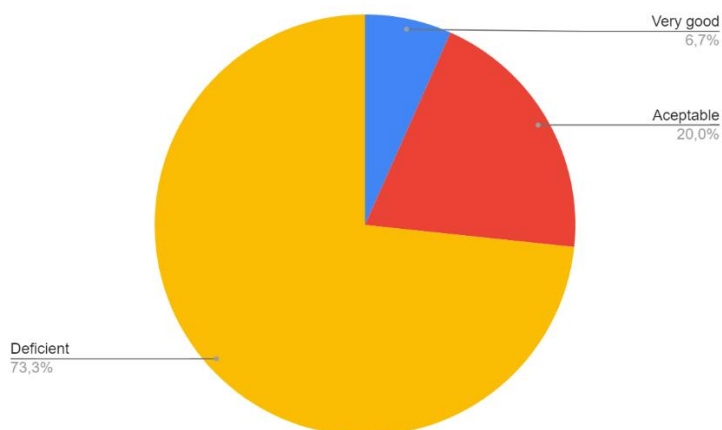
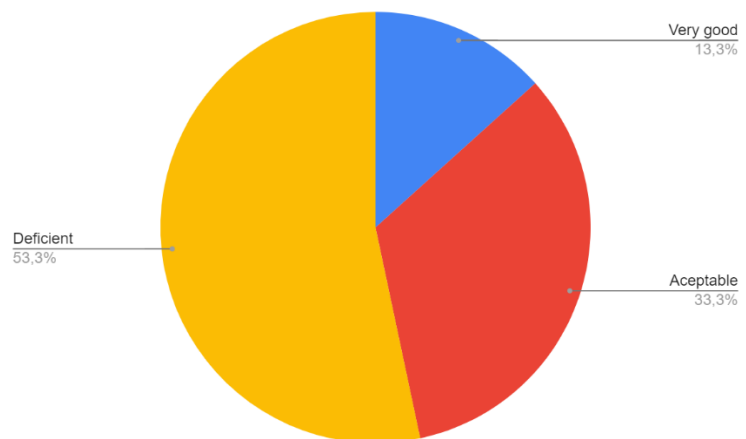
Figure 12*Participants' Self-reported English-Speaking Proficiency*

Figure 12 shows the participants' self-perceived English writing proficiency. Most participants rated their writing skills as deficient and only two as very good. Even though this is a skill the participants did show much interest in improving, it is still considered a lack and commonality of the learner population. In addition, Figure 13 shows almost the same percentages regarding their English listening proficiency, where more than half of the participants considered their listening skills as deficient and only three considered them as acceptable. Finally, Figure 15 shows English reading proficiency as the skills that participants struggle with the least. Less than one third of the participants rated their reading skills as deficient. These data confirm the information collected from the initial interview, namely the stakeholders' statement that the applied and pure mathematicians have stronger English reading and writing skills since everything they read is mostly in English and they also conduct research mostly in English.

Figure 13

Participants' Self-reported English Writing Proficiency

**Figure 14**

Participants' Self-reported English Listening Proficiency

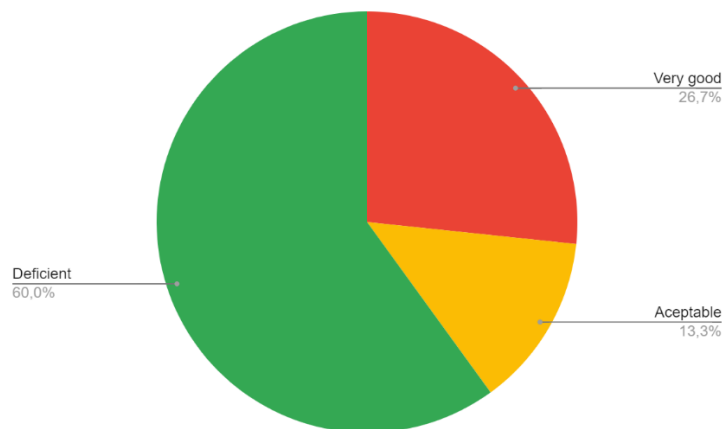
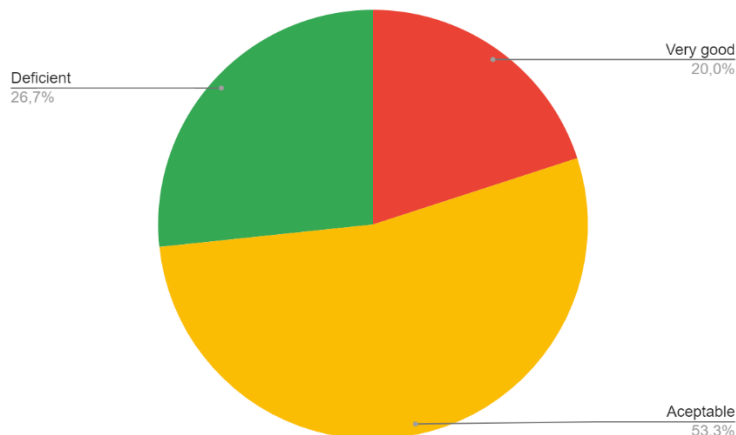
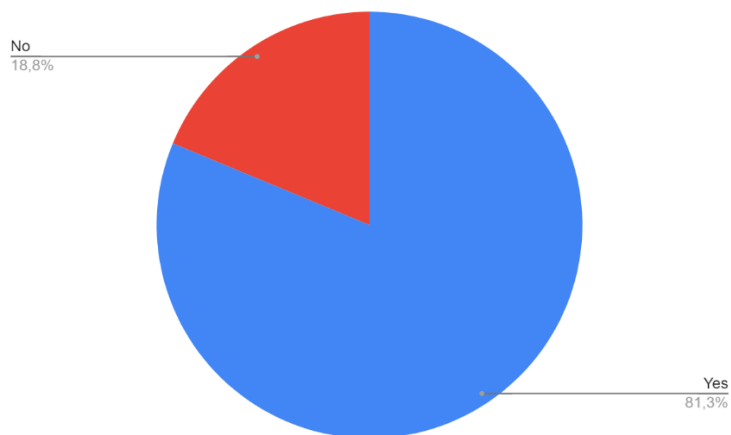


Figure 15*Participants' Self-reported English Reading Proficiency****Students' Learning Strategies, Learning Styles, and Attitudes***

As shown in Figure 16, two thirds of the participants indicated that they enjoy taking English classes. Even though it might be challenging for most of the participants to learn English for teaching mathematics, these data show positive numbers regarding the participants' attitude towards learning the target language. This is relevant considering that students' attitude and motivation can play a key role in the language learning process. Figure 18 shows that, when asked whether they learn English as an obligation and not as a personal interest, most participants do not see it as an obligation. Figure 18 shows that the majority of the participants also claimed to enjoy being in contact with the language outside of class including watching movies, listening to music, and reading. Finally, Figure 19 shows that most participants have a positive attitude towards the idea of integrating English into their math lessons compared to only two who stated the opposite.

Figure 16

Enjoy Taking an English Course

**Figure 17**

Learn English as an Obligation and Not as a Personal Interest

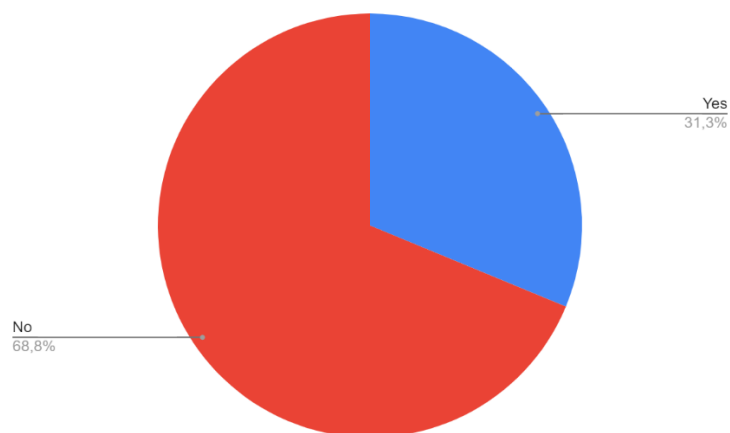
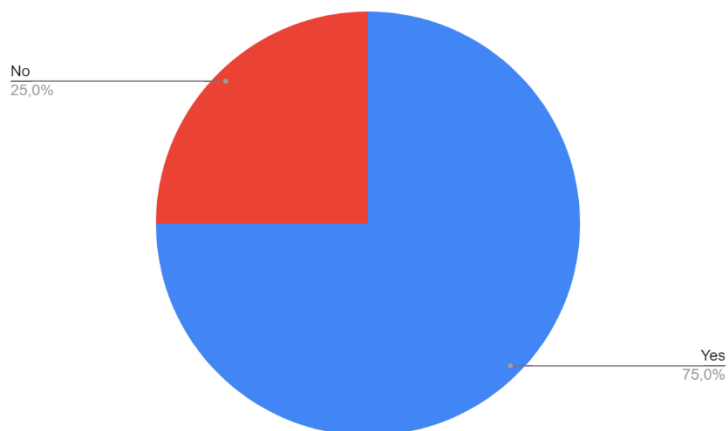
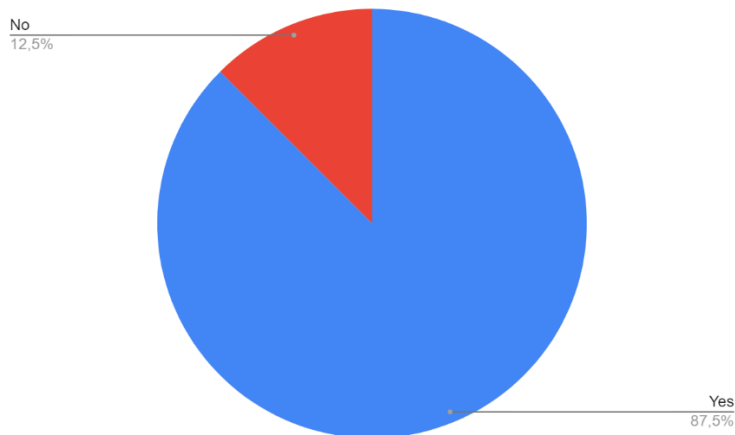


Figure 18

Enjoy Being in Contact with the Language Outside of Class (movies, music, books)

**Figure 19**

Like the Idea of Integrating English into Math Lessons



Diagnostic Test

Test Administration Issues

As previously stated, the course-design team contacted the Director of the Mathematics Department (POC) via email to schedule a Zoom session with all the participants to administer the diagnostic test to all participants. The POC then contacted

the participants to coordinate the date and time for the administration of the test. The time allotted for the listening and reading sections was 60 minutes. The course-design team began the session by explaining the purpose of the diagnostic test followed by reading the instructions of the test together, while allowing time for participants to ask questions as needed. After that, individual appointments were scheduled with the participants to carry out the oral section of the test. Each individual appointment took about 15 minutes. The course-design team started each individual appointment with a warm-up question that aimed to put students at ease followed by providing instructions and modeling the task in Spanish to clarify what was expected of participants when performing the oral task.

Diagnostic Test Design

Macro Skills and Micro Skills. Based on the data collected in the initial semi-structured interview with stakeholders and the needs analysis survey for prospective students, the macro skills selected to design the diagnostic test were reading, listening, and speaking. This corresponds to the key task that emerged from the data analysis: explaining mathematical concepts and procedures orally. Writing was the only macro skill not included in the diagnostic test because writing skills were not part of the prospective students' reported needs and wants for an ESP course. The participants' main goal is to produce the target language in oral form rather than in writing, and they expect a strong focus on mathematics vocabulary and pronunciation as a result. Since reading and listening to authentic texts related to the desired output in oral tasks (explaining concepts and procedures, asking and answering questions) can serve as a rich source of vocabulary, these two macro skills are part of the selection. With a focus on the same tasks, the micro skills selected for the diagnostic test included relevant vocabulary. Other micro skills selected for the diagnostic test included reading and listening for specific details and fluency.

Listening. To assess listening skills, the first part of the test consisted of two listening tasks. The first task elicited a productive response and tested listening for specific details as a micro-skill. The students had to listen twice to an explanation about how to add and subtract fractions to fill in the blanks with key mathematical vocabulary related to fractions. The purpose of this task was to determine if students were able to recognize basic mathematical terms in English, which is suitable for a basic-user level of proficiency according to the Common European Framework of Reference for Languages (CEFR). The second task also tested listening for specific details (target micro-skill) but elicited a receptive response. The students had to listen twice to an explanation on finding common denominators to organize a sequence of steps in the right order according to the information presented in the audio. The purpose of this task was to determine if students were able to arrange a sequence of steps in a process by recognizing and understanding discourse markers of sequence in addition to relevant mathematical terminology in English. Therefore, this task tested comprehension, which is appropriate for an independent-user level of proficiency according to the CEFR. These listening tasks contained the constructs of vocabulary and comprehension, specifically listening for specific information.

Reading. To assess reading skills, the second part of the test consisted of two different reading tasks. The purpose of the first task was vocabulary recognition, appropriate for basic users according to the CEFR. After reading the input, students had to demonstrate understanding of basic mathematical vocabulary in the expressions by selecting the correct formulation. For this task, students first needed to understand every single word of the formulations available as options so that they could select the one that correctly represented the expression. Therefore, it reflected the construct of vocabulary recognition. For the second task, students had to demonstrate comprehension by organizing a procedure in a logical order, suitable for an intermediate level of proficiency.

The students first needed to understand the meaning and the content of a text and then understand more specific parts of the text so that they could arrange them chronologically. Thus, the task included the construct of reading comprehension, mainly reading for specific details.

The assessment criteria for both listening and reading tasks was based on parameters that evaluate performance based on the number of correct answers rather than on a given rubric to claim whether the students were successful at completing the tasks or not. This is because the scoring process for these tasks was framed into a discrete-point approach, where the answers were either correct or incorrect. For that reason, a rubric would not have been useful or necessary since scoring was already objective enough (either right or wrong answers). This objectivity was attained in the first listening and reading tasks by having the students identify elements “isolated from a continuous stream of speech” (Buck, 2001, p.66) and in the second listening and reading tasks by organizing a series of steps based on specific discourse markers and key mathematical vocabulary in English. Although the primary purpose of this diagnostic test was to test language through tasks with increasing proficiency levels, testing for advanced listening and reading micro-skills like making inferences was beyond its scope because the students self-reported having a basic or intermediate level of linguistic proficiency in the needs analysis survey.

Speaking. To assess speaking skills, the third part of the test consisted of three speaking tasks. These tasks did not seek to assess specific communication skills like providing a step-by-step explanation but aimed to evaluate a limited number of language criteria instead. These included vocabulary and pronunciation, fluency, and context-specific structures. That is, the focus of these tasks was on specific outputs that were not always communicative in nature, but included basic components of the oral tasks that the students need to perform, for example, reading mathematical operations out loud and

correctly sequencing the steps of a mathematical procedure. This last part of the test was carried out on a one-on-one basis, following a fifteen-minute appointment schedule. A quick warm-up question was asked prior to providing instructions for the speaking tasks.

These speaking tasks were scaffolded with an increasing level of difficulty. For the first task, the students were provided with nine operations and equations written in mathematical symbols, which they had to read aloud one by one. This task tested at the basic-user level, as its focus was on correct use of vocabulary, intelligible pronunciation, and proper fluency. For the second task, the students were provided with three solution procedures and the corresponding sequence of mathematical operations required solving an equation (middle-school level). The students were then asked to read each step or mathematical operation out loud (also written in mathematical symbols) using appropriate vocabulary and sequencing language, along with any other relevant structures such as modal verbs. This task tested at the independent-user proficiency level, as the students had to explain various steps in a process. The third task was designed to test for more advanced proficiency levels. For this task, the students were asked what mathematics topics have been the most difficult for them to learn and to teach and why. The purpose of this last task was to confirm if any of the students would be able to explain or justify their opinion using specialized and more advanced vocabulary, while staying on the topic of mathematics. As noted above, the speaking tasks contained the constructs of vocabulary, pronunciation, fluency, and structure.

The diagnostic test was thus designed with the basic and independent proficiency levels in mind. As mentioned above, the listening and reading sections of the test included two tasks each, the first one intended for basic users and the second one for independent users. The speaking part included three tasks intended for basic, independent, and proficient levels (one task per level). The purpose of this increased level of difficulty of the third speaking task was to assess communicative proficiency, fluency, and the language

skills the target population needs to develop according to the data collected from the needs analysis survey. The criteria that the CEFR provides to measure speaking, writing, reading, and listening skills were considered to classify the proficiency levels of the tasks included in the diagnostic test. As previously stated, the test did not include writing tasks since the learner population does not need to engage in writing tasks of any kind.

Diagnostic Test Results

Listening. In the listening section, the majority of the students were partially successful or not successful at all at completing the listening tasks based on the number of correct and incorrect answers gathered. The detailed results are presented in Table 2.

Table 2

Listening Section, Task 1 Result

Statements	Correct answers	Number of participants who answered correctly	Number of participants who answered incorrectly
1. The word used in the audio to describe the numerator of a fraction is _____ number.	top	5	10
2. The word used in the audio to describe the denominator of a fraction is _____ number.	bottom	7	8
3. The word used in the audio to name fractions with the same denominators is _____ fractions.	like	7	8

Statements	Correct answers	Number of participants who answered correctly	Number of participants who answered incorrectly
4. The word used in the audio to name the fractions with different denominators is _____ fractions.	unlike	6	9
5. The expression used in the audio to describe the process of changing fractions to add them is called _____.	finding a common denominator	6	9

Note. Based on the results of Task 1 of the diagnostic test.

Table 2 presents the results obtained after the administration of the first listening task. As a guide, it also displays the statements that the participants had to complete when listening to the audio, as well as the correct answer in each case. Only 3 out of 15 participants were able to fill in the blanks with the missing information correctly. The other 12 participants showed difficulty in recognizing and completing the statements with the correct word. As can be seen, more incorrect answers were obtained for each of the sentences, especially in the recognition of basic words in fractions such as top, bottom, like, and unlike when listening to an explanation of a common procedure. These results indicate that even though Task 1 was designed for a basic level of performance, over half of the participants failed to identify basic mathematics vocabulary as part of the listening skills that they need to develop for their mathematical instruction fields, and only three of them successfully completed the task indicating a higher performance in the language.

Table 3*Listening Section, Task 2 Results*

Statements	Correct answers	Number of participants who answered correctly	Number of participants who answered incorrectly
1. Continue making multiples until you find the same multiple (or a common multiple) for both denominators. -	3	13	2
2. Multiply the fractions to get new equivalent fractions with the same denominator.	5	8	7
3. Start making multiples of each denominator.	1	12	3
4. Create a small table to organize the multiples of the two denominators.	2 (answer provided to participants)	15	0
5. Add the numerators and keep the common denominator to add the fractions.	6	8	7
6. Find the whole fractions that should be multiplied by the original fractions.	4	7	8

Note. Based on the results of Task 2 of the diagnostic test.

Table 3 presents the results obtained after the administration of the second listening task. As can be seen, the table displays a set of procedures that the participants

had to organize correctly when listening to the audio, as well as their correct answers. For this task, the level of difficulty was higher where the aim was not only the understanding of the explanation of the procedure, but also the correct organization of each step. The results indicate that 6 out of 15 participants successfully identified the logical sequence of the steps provided in the task. The other 9 participants found it difficult to determine the proper order of the steps. It should be noted that step 2 was provided to the students in order to support them in understanding the task. Therefore, over half of the participants are considered as low performers while the other 9 are better performers in terms of listening skills in the language.

Reading. In the reading section, all of the participants were able to recognize basic mathematical vocabulary in the expressions by always selecting the correct formulation in a multiple-choice task. The data suggests that participants were able to perform at a level corresponding to a basic user in the first reading task because they “can deduce the meaning of a word from an accompanying picture or icon” (Council of Europe, 2018, pg. 67). Table 4 shows that students were able to identify target vocabulary such as *less than*, *subtract from*, *divide by*, *multiply by*, and *add* by selecting the corresponding mathematical expression.

Table 4

Reading Section, Task 1 Results

Questions	Correct answers	Number of Participants Who Answered Correctly	Number of participants Who Answered Incorrectly
1. Which expression means 6 less than 15?	15 – 6	15	0

Questions	Correct answers	Number of Participants Who Answered Correctly	Number of participants Who Answered Incorrectly
2. Which expression represents “subtract 6 from 18, then divide by 3”?	$(18 - 6) \div 3$	15	0
3. Which expression means “add 12 and 9, then multiply by 6”?	$6 \times (12 + 9)$	15	0
4. Which expression represents “add 465 and 371, then multiply by 2”?	$2 \times (465 + 371)$	15	0
5. Rachel bought and downloaded 6 songs on Friday and 8 more songs on Saturday. Each song cost \$2. Which expression represents the amount of money that Rachel paid?	$2 \times (6 + 8)$	15	0

In the second task of the reading section, only 2 out of 15 participants were able to organize the steps in the mathematical procedure in logical order from 1-7. The other 13 participants organized some of the steps correctly, but other steps were organized incorrectly. The data suggest that the performance of only two participants is comparable to a B1 because they were able to “extrapolate the meaning of a section of a text by taking into account the text as a whole” (Council of Europe, 2018, pg. 67). Table 4 shows that most participants failed steps 2 and 4. These two procedures do not have any logical

connectors at the beginning of the statement, which requires a higher level of reading comprehension that only independent users are more likely to possess.

According to the CEFR, independent users “can follow a line of argument or the sequence of events in a story, by focusing on common logical connectors (e.g., however, because) and temporal connectors (e.g., after that, beforehand)” (Council of Europe, 2018, pg. 67). This descriptor matches the performance of the other 13 participants who partially organized the steps of the mathematical procedure in logical order. Table 5 shows that most participants organized step three, seven, and five successfully, precisely those that start with logical connectors such as “In the warm-up”, “The next lesson”, and “Finally”. The implication is that most of the group is at a level where they can organize steps when provided with explicit markers, with a task performance corresponding to the level in question.

Table 5

Reading Section, Task 2 Results

Statements of Mathematical Procedure about the Volume of a Cylinder	Correct answers (step #)	Number of Participants Who Answered Correctly	Number of Participants Who Answered Incorrectly
In the warm-up, students recall that a circle’s area can be determined given its radius or diameter.	3	7	8
The next lesson will provide opportunities to practice these skills and solve related problems.	7	14	1

Statements of Mathematical Procedure about the Volume of a Cylinder	Correct answers (step #	Number of Participants Who Answered Correctly	Number of Participants Who Answered Incorrectly
In this lesson students learn that the volume of a cylinder is the area of the base times the height, just like a prism.	1	(answer provided to participants)	0
Finally, students compute the volume of a cylinder by multiplying the area of its base by its height.	5	8	7
This is accomplished by considering 1-unit-tall layers of a rectangular prism side by side with 1-unit-tall layers of a cylinder. After thinking about how to compute the volume of specific cylinders, students learn the general formulas $V=Bh$ and $V=\pi R^2h$.	2	3	12
A volume expressed using the exact number π versus the same volume computed using 3.14 as an approximation for π is discussed.	6	(answer provided to participants)	0
Students also become familiar with what is meant by <i>radius</i> and <i>height</i> as those terms apply to cylinders.	4	3	12

Table 6*Summary of Performance Levels in the Reading and Listening Tasks*

Macro-Skill	Top Performers (10-12 correct answers)	Intermediate Performers (7-9 correct answers)	Low Performers (0-6 correct answers)
Listening	2/15	4/15	9/15
Reading	4/15	11/15	0/15

Table 6 shows a brief summary of performance levels in the reading and listening tasks based on the number of correct answers on each macro-skill. As can be seen, in the listening tasks, the vast majority of participants are low performers with just a few as top and intermediate performers. In the reading tasks, the vast majority are intermediate performers with just a few as top performers, and none as low performers.

Speaking. The speaking section of the diagnostic test was administered to all 16 prospective students (one of them was not present during the administration of the listening and reading sections of the test due to a family commitment, but she was able to join for the speaking section). In this section, the majority of the students were partially successful or not successful at all at completing the speaking tasks based on the parameters in Table 5 below.

Table 7*Students' Performance in the First Two Speaking Tasks*

Parameters	Successful		Partially Successful		Unsuccessful	
	<i>n</i>	%	<i>n</i>	%	<i>n</i>	%
Pronunciation	2	12	8	50	6	38
Vocabulary	1	6	7	44	8	50
Fluency	1	6	5	32	10	62
Sequence markers	2	12	6	38	8	50

Note. Pronunciation and vocabulary refer mostly to numbers, symbols, and operators.

Pronunciation. Only two out of the 16 students were able to pronounce most numbers correctly. Although nearly all students demonstrated that they knew the numbers in English, they did not pronounce some of them correctly. Remarkably, three students did not even know how to say some of the numbers in English, particularly numbers higher than 10, so they either filled in the gaps with the numbers in Spanish or simply skipped them while attempting to read the operations. The same students still mispronounced most of the numbers they did know in English. Only the two students who pronounced most numbers correctly were also able to pronounce mathematical symbols and arithmetic operators correctly. Nearly half of the students were able to pronounce most of the symbols and operators correctly, while the other half did not know how to say most of them in English.

Vocabulary. Half of the students were not able to read most of the operations in the first two speaking tasks, suggesting that they have very limited vocabulary. As for the other half, only one student was able to use most relevant vocabulary correctly in the first two speaking tasks, while the rest was partially successful at reading the operations due to some gaps in their vocabulary inventory. Overall, the results show that the vast majority of

the students do not master the basic vocabulary related to operations, equations, and procedures.

Fluency. It is not surprising that, considering the significant pronunciation and vocabulary gaps and deficiencies that the majority of the students demonstrated, only one of them was successful at reading the operations in Task 1 and explaining the procedures in Task 2 fluently. Over half of the students were not successful at reading the operations in the first task, while two thirds of the students were not successful at explaining the procedures in the second one. That is, nearly all students lacked fluency and confidence when reading operations and explaining procedures, as shown by their hesitant speech and continuous pausing.

Sequence markers. The correct use of sequence markers was one of the micro-skills tested in the second task. For this task, only two out of the 16 students managed to use basic sequencing language correctly and consistently. Half of the students were able to explain the various steps in a mathematical procedure but use appropriate sequencing language occasionally, while the rest of the students used only a limited number of discourse markers of sequence or none at all.

Discussion of Results

The data collected through the needs analysis instruments confirm that speaking is the main macro skill participants want and need to develop in the future ESP course. Furthermore, these data suggest that speaking in English, particularly explaining mathematical concepts and procedures, is a daunting task for most of them. In fact, the majority of them were not able to complete the speaking tasks of the diagnostic test thoroughly and successfully based on the parameters outlined above. In other words, most of them were not able to use relevant vocabulary and discourse markers properly, confidently, and fluently. What is more, half of them can be regarded as low performers because they were not able to successfully complete any of the speaking tasks. According

to the *Qualitative Aspects of Spoken Language Use* of the CEFR, the speaking performance of most students on this test is comparable to that of an A1 because they can manage very short, isolated, mainly pre-packaged utterances, with much pausing to search for expressions, to articulate unfamiliar words, and to repair communication. This means that the course-design team needed to start the ESP course by exposing the learner population to as much target input as possible so that they first get familiar with basic terminology and structures, which in turn would allow them to explain mathematical concepts and procedures properly, confidently, and fluently. Even if they do not consider listening and reading as relevant macro skills, they needed to receive abundant input through listening and reading necessary for building accuracy, confidence, and fluency when performing the target speaking tasks.

Chapter II. Syllabus Design

This chapter presents the Syllabus Design report as submitted prior to its approval for implementation in the 2nd semester of 2023.

Course Logo



The course logo depicts mathematics as the science that deals with the logic of shape, quantity, and arrangement. Mathematics is present in a wide variety of processes that are carried out on a daily basis. For this reason, it was decided to incorporate the main and most well-known symbols of mathematics in the course logo (addition, subtraction, division, and multiplication). As for the name of the course, *Limit Breakers English Club* represents the beginning of a challenge towards bilingual mathematics education that aims to achieve the personal, academic, and professional goals of each individual.

Course Description

This course was taught by three practicum students of the Master's Program in English Teaching as a Foreign Language to the professors of the Mathematics Department at UCR's Western Campus (Sede de Occidente). This course is designed with the purpose of providing students with sufficient input in the macro skills of listening,

reading, and speaking. Likewise, it is intended to incorporate vocabulary, pronunciation, fluency, and structure as part of the micro skills related to the area of mathematics teaching.

The group is composed of 16 students, whose estimated level of English proficiency in field-related tasks ranges from low to intermediate performers according to the diagnostic test previously administered as part of the needs analysis process. The course was held on Tuesdays from 5:00 p.m. to 7:00 p.m. virtually and lasted for 16 weeks, for a total of 48 hours. The course was delivered synchronously through Zoom and asynchronously through a variety of digital tools during the second semester of 2023. Regular attendance and active participation was required of students to achieve the proposed objectives and pass the course. *Limit Breakers English Club* obeys to the type of syllabus, which is task-oriented, where meaning and communication are the main focus along with academic, professional, and real-life tasks (see appendix D).

Statement of Goals and Objectives

Unit 1: Delivering Arithmetic, Algebra, and Geometry Input

Goal: At the end of the unit, students will be able to create materials in preparation for arithmetic, algebra, and geometry lessons aimed at bilingual mathematics teachers by identifying key vocabulary and expressions while listening to and reading authentic input in the target language.

General Objectives: By the end of the lesson, students will be able to:

1. effectively compile a glossary of basic vocabulary and expressions related to arithmetic, algebra, and geometry by listening to explanations and descriptions of mathematical procedures.
2. successfully expand a glossary of basic vocabulary and expressions with more advanced terms related to arithmetic, algebra, and geometry by reading explanations and descriptions of mathematical procedures.

3. create a simple teaching resource to explain the pronunciation of basic vocabulary and expressions related to arithmetic, algebra, and geometry by using digital tools for listening, repeating, and reading aloud.

Unit 2: Delivering Arithmetic, Algebra, and Geometry Lessons

Goal: At the end of the unit, students will be able to explain basic mathematical procedures related to arithmetic, algebra, and geometry by using the correct vocabulary and expressions, modal verbs, sequencing markers, and intelligible pronunciation.

General Objectives: By the end of the lesson, students will be able to:

1. successfully identify target vocabulary necessary for arithmetic, algebra, and geometry lessons by analyzing written and audio passages containing explanations of mathematical procedures.
2. accurately organize the steps of mathematical procedures in the correct sequence in spoken form by using sequencing words and modal verbs.
3. accurately describe mathematical procedures orally by using previously learned vocabulary, appropriate sequencing words and modal verbs, and intelligible pronunciation.
4. effectively explain mathematical procedures related to arithmetic, algebra, and geometry by simulating a mathematics teacher describing procedures in an oral presentation in class.

Unit 3: Delivering Basic Geometry and Trigonometry Lessons

Goal: At the end of the unit, students will be able to adequately deliver basic geometry and trigonometry lessons by explaining introductory level examples and problems using the corresponding vocabulary and sequencing markers.

General Objectives: By the end of the lesson, students will be able to:

1. effectively identify basic vocabulary and sequencing markers necessary for geometry lessons by completing listening and reading exercises based on authentic texts.
2. effectively identify and use basic vocabulary and sequencing markers necessary for basic trigonometry lessons by completing listening and reading exercises based on authentic texts.
3. successfully deliver basic geometry and trigonometry lessons by using the correct vocabulary, sequencing markers, and modal verbs.

Methodology

This course allows students to integrate reading, listening, and speaking into their academic, professional, and real-life needs. Those needs are reflected in different tasks developed in three units, which are taught by three instructors. Lessons will be focused on communication and meaning, allowing students to clarify information with their classmates and instructors, recognize and apply technical vocabulary in different contexts, learn the pronunciation of new words, comprehend texts, and develop successful listening skills in mathematics instruction. Instructors will use authentic material such as texts, audios, worksheets, and pictures.

Approach

The course is based on the Task-based Language Teaching (TBLT) approach for its usefulness and ease of language learning. This approach meets the specific needs of the course, which has the specific purpose of introducing the teaching of mathematics in the English language. As Norris (2002) states,

Task-based language teaching (TBLT) is an approach to second or foreign language education that integrates theoretical and empirical foundations for good pedagogy with a focus on tangible learning outcomes in the form of “tasks” – that is, what learners are able to do with the language. (p. 578)

This definition refers to the need to justify the design of a course with a variety of opportunities for students to not only understand the language for specific purposes, but also to be part of real-life communicative experiences. These communicative experiences emphasize the effective performance of a task that requires an explicit context and a high level of participation on the part of those involved. In other words, TBLT addresses the requirements of diverse language learning contexts and acts on the demands of language learning diversity in the educational scope, providing educators with the opportunity to design effective and meaningful programs according to the requirements and desires of the learners making it a suitable approach for an ESP course focused on developing language skills to perform real-life field-related tasks.

Classroom Dynamics

The instructors developed the lesson plans together for each week. However, they rotated every week to be the lead instructor while the other two acted as assistants. The role of the lead instructor is to carry out the lesson plan, monitor students' performance, and clarify as needed. The assistant instructor monitored students' work, organizing and setting up the material for the activities, modeling conversations with the lead instructor, and providing other types of support when necessary. In addition, assistant instructors took notes on different aspects of teaching and classroom dynamics as well as students' performance to provide feedback and collect evidence for course improvement.

Tasks and Techniques

"Tasks are activities where the target language is used by the learner for a communicative purpose (goal) in order to achieve an outcome". (Willis, 1996, as cited in Willis & Willis, 2007, p. 12). With this in mind, students performed tasks such as role-plays between a math teacher and a student, oral presentations where students were

requested to explain mathematical procedures, as well as reading and listening tasks that exposed them to relevant mathematical terms.

Purposeful grouping techniques were used to pair or group students with varying skill levels. In addition, Kagan's (2009) cooperative learning structures such as Rally Robin, Timed Pair Share, Round Robin, and Rally Coach were implemented along with the tasks.

Role of the Learner

According to Richards and Rodgers (2001), students have three main roles in the TBLT approach. The first consists of being a negotiator of meaning. In other words, students played an active role participating in communicative tasks in pairs or groups, asking questions, and clarifying or rephrasing information. In addition, "many tasks required learners to create and interpret messages for which they lack full linguistic resources and prior experience" (Richards & Rodgers, 2001, p. 235). Therefore, students can also take the role of risk-takers and innovators since they are exposing themselves to something new and challenging in every class. Finally, students played the role of self-monitors when they are encouraged to monitor their own progress. This last role fostered learners' awareness of their own needs and strengths with the language.

Role of the Instructors

The instructors played different roles including being a creator, a facilitator, and a motivator (Willis, 2007). They played the role of creators since they must design and/or adapt the instruments, tests, course syllabus, and materials. In addition, the instructors are facilitators of learning. They need to provide learners with the proper tools, strategies, techniques, and consistent guidance that students need in order to learn the target language successfully. Finally, the instructors played the role of motivators. Learning a second language is very challenging for most students. Therefore, having instructors that

encourage learners whenever they are feeling overwhelmed or disappointed is essential for them to succeed.

Assessment

Assessment is an ongoing process that should combine a variety of methodological techniques. According to Brown (2019), a good teacher should assess students' performance throughout the course and not just at the end of a lesson, unit, or term of study. For this reason, informal assessment is inextricably interwoven in the course design and delivery. This informal assessment took different forms, primarily that of incidental, unplanned, nonjudgmental comments and responses from the instructors. This means that informal assessment was embedded in the various classroom tasks and activities, which were designed to elicit performance without "recording results and making fixed conclusions about a student's competence" (Brown, 2018, p. 7). For example, impromptu, targeted feedback was provided while or after completing a task to give students an idea of what they did well, in the form of praise, and what they need to improve, in the form of corrective feedback. Students also received ongoing advice about how to better pronounce words and how to memorize and use new vocabulary items in context.

Naturally, systematic, planned tasks and exercises intended to formally appraise students' competence and performance also was an integral part of the course design. As part of this formal assessment, students took a series of short listening and reading comprehension tests throughout the course and an oral exam at the end of the program. The primary goal of this final *achievement* test is to determine whether the general course objectives have been met by the end of the period of instruction. In addition to playing a summative role, this final exam also played a formative role because it offered valuable and actionable insights and feedback for a potential, follow-up course. However, "not all formal assessment is testing" (Brown, 2018, p. 7), so students also created a

glossary and undertake a term oral project for which they prepared and recorded a ten-minute teaching demonstration that helped them to attain some of the course objectives. Therefore, a grade obtained in each of these planned evaluative activities counted towards their final grade in the course.

In addition to summative assessment, formative assessment also played a vital role in the course design and delivery. Brown (2018) defines formative assessment as the one that evaluates students in their “process of forming their competencies and skills with the goal of helping them to continue that growth process” (p. 6). Most classroom assessment is in fact formative assessment, as all types of informal assessment have as “their primary focus the ongoing development of the learner’s language” (p. 8). As stated above, students were constantly provided with targeted, non-evaluative feedback that helped them to hone their language skills, identify their strengths and weaknesses, and guide their own learning (Andrade & Cizek, 2010). Furthermore, another benefit of formative assessment is that it can assist the instructors in the planning of subsequent lessons by providing valuable insight into the target forms and structures that the learners need more practice in (Andrade & Cizek, 2010).

Assessment Tasks

Table 8 includes the evaluation distribution for this course.

Table 8

Course Evaluation

Assessment	Value
Listening comprehension quizzes (3)	15%
Reading comprehension quizzes (3)	15%
Course glossary	10%
Attendance	10%

Assessment	Value
Final Oral Exam	25%
Term Oral Project	25%
Total	100%

Listening Comprehension Quizzes (3). To assess listening skills throughout the course, the students completed three short listening assessment tasks, 5% each, one per unit. The listening performance for these assessment tasks were both intensive (listening for perception of the components, including vocabulary, discourse markers, and modals) and selective (listening for specific information), according to Brown's (2018) categories of listening assessment tasks. The purpose of these tasks is to determine if students are able to recognize the mathematical terms, sequencing markers, and modal verbs covered in each unit. They may not be communicative or interactive tasks in nature, but they are cognitively demanding and based on authentic listening passages and on tasks carried out in class.

Reading Comprehension Quizzes (3). To assess reading skills throughout the course, the students completed three short reading assessment tasks, 5% each, one per unit. The ultimate goal of these tasks is vocabulary recognition, specifically in context. After reading the passages, students have to demonstrate understanding of relevant mathematical terms and structures. According to Brown (2018), this assessment task can be considered a perceptive reading task because it involves attending to the components of larger stretches of discourse, so bottom-up processing is required. In addition, students also had to demonstrate comprehension by organizing mathematical procedures in logical sequencing, so they first need to understand the meaning and the content of a text and then understand more specific parts of it; top-down processing is thus required.

Consequently, a combination of bottom-up and top-down processing was required to complete these reading assessment tasks.

Course Glossary. This is a form-focused type of assessment (Brown, 2018). For this task, students created an alphabetical list of 30 mathematical terms and definitions, including Spanish equivalents, example sentences with the terms used in context, and a quick reference guide to their pronunciation in English. The purpose of this summative assessment task is not only to help students recognize these terms when they see or hear them but also to be able to put vocabulary knowledge to use in a given context by providing an example sentence for each term. According to Nation (2013), using a word in speaking or writing requires a different, and perhaps more complex, set of abilities from those needed for simply understanding a word in context, so both “receptive” and “productive” vocabulary knowledge is considered in this formal assessment task.

Attendance. To recognize and encourage class attendance, the course-design included course attendance as part of the course evaluation. It is understandable (and to some extent expected) that students may want to miss class occasionally because the course may be perceived as too challenging, the students may feel overwhelmed by other commitments, or they may feel too tired for evening classes. Even if attendance is mandatory and a certain number of excused absences may be allowed, students should aim to attend every class session simply because class attendance and participation can enhance their language competence and performance in multiple ways. That is why, at the end of the program, 10% of the course evaluation was given to those students who never missed class sessions, and a fraction of that percentage was taken off for every class session missed. The instructors created an attendance tracker to log student attendance.

Final Oral Exam. The final oral exam aims to measure the students’ speaking competence in English. For this exam, students were provided with solution procedures that include sequences of mathematical operations required to solve different equations.

They then were asked to read each step or mathematical operation out loud (also written in mathematical symbols) using appropriate vocabulary and sequencing language, along with any other relevant structures such as modal verbs. Brown (2018) refers to this type of task as “extensive speaking,” as opposed to “interactive speaking,” where the difference between these two kinds of tasks is “the degree of interaction with an interlocutor” (p. 178). This oral production assessment task mirrors the main speaking task of the diagnostic test, which allowed the instructors to compare results before and after course completion. To complete this “transactional” speech event, students were also assigned an appointment with at least one of the instructors and was also recorded for future reference and feedback. As can be seen, this is a summative, task-based type of assessment. According to Brown (2005), this type of assessment is a subset of performance-based assessments because the focus is explicitly on “particular tasks or task types” in a curriculum (p. 24). Given that this task-based exam involves learners in performing a specific target behavior, it links classroom activities to real-world tasks (Brown, 2018).

Term Oral Project. The term oral project also aims to measure the students’ speaking competence in English. For this project, students prepared and record a ten-minute teaching demonstration in which they explained how to solve a series of basic mathematical operations and equations provided by the instructors. The obvious difference between this summative assessment and the final exam is that students recorded themselves completing this task at their own convenience, which allowed them to check vocabulary items, pronunciation, and target structures as needed prior to recording themselves. This is another task-based assessment, which is in turn also a subset of performance-based assessments. It focuses on a particular task that approaches the authenticity of real-life language use for the students, so it also links classroom activities to real-world tasks (Brown, 2018). Just like the final oral exam, this oral assessment illustrates an extensive, transactional, real-world speech event, which is explaining, step

by step, mathematical operations and equations. Since language competence in interpersonal or social speech events is out of scope for this course, no interactive, communicative tasks are included as part of the summative oral assessment tasks. That is why a checklist instead of a rubric was used to validate the specific, target parameters of the students' oral performance for both the final exam and the term project.

Contents

This course consists of three units, which were developed to fulfill the students' needs, wants and lacks. Each unit has three objectives with one task each, and instructors used a variety of skills, language focus, and strategies as seen below.

Unit 1: Delivering Arithmetic, Algebra, and Geometry Input

Goal: At the end of the unit, students will be able to create materials in preparation for arithmetic, algebra, and geometry lessons aimed at bilingual mathematics teachers by identifying key vocabulary and expressions while listening to and reading authentic input in the target language.

General Objectives: By the end of the lesson, students will be able to:

1. effectively compile a glossary of basic vocabulary and expressions related to arithmetic, algebra, and geometry by listening to explanations and descriptions of mathematical procedures.
2. successfully expand a glossary of basic vocabulary and expressions with more advanced terms related to arithmetic, algebra, and geometry by reading explanations and descriptions of mathematical procedures.
3. create a simple teaching resource to explain the pronunciation of basic vocabulary and expressions related to arithmetic, algebra, and geometry by using digital tools for listening, repeating, and reading aloud.

General Objective	Tasks	Skills	Language Focus	Strategies	Time allotted
1	Listen to explanations and descriptions of mathematical procedures to compile a glossary of basic technical vocabulary.	L	Vocabulary Mathematical terms and expressions in the areas of arithmetic, algebra and geometry. Examples: addition, subtraction, division, multiplication, plus, minus, divided by, times, variable, equation, algebraic expression	Identifying words and phrases Taking notes Checking understanding	1 lesson plan
2	Read explanations and descriptions of mathematical procedures to compile a glossary of basic technical vocabulary.	R	Vocabulary Mathematical terms and expressions in the areas of arithmetic, algebra and geometry. Examples: addition, subtraction, division, multiplication, plus, minus, divided by, times, variable,	Skimming Scanning Small group sharing	1 lesson plan

			equation, algebraic expression		
3	Produce the correct pronunciation of key vocabulary and expressions in the field of mathematics.	L R	Vocabulary Mathematical terms and expressions in the areas of arithmetic, algebra and geometry. Examples: addition, subtraction, division, multiplication, plus, minus, divided by, times, variable, equation, algebraic expression	Repeating Reading aloud Identifying pronunciation Producing accurate pronunciation Checking intelligibility	1 lesson plan

Unit 2: Delivering Arithmetic, Algebra, and Geometry Lessons

Goal: At the end of the unit, students will be able to explain basic mathematical procedures related to arithmetic, algebra, and geometry by using the correct vocabulary and expressions, modal verbs, sequencing markers, and intelligible pronunciation.

General Objectives: By the end of the lesson, students will be able to:

1. successfully identify target vocabulary necessary for arithmetic, algebra, and geometry lessons by analyzing written and audio passages containing explanations of mathematical procedures.
2. accurately organize the steps of mathematical procedures in the correct sequence in spoken form by using sequencing words and modal verbs.
3. accurately describe mathematical procedures orally by using previously learned vocabulary, appropriate sequencing words and modal verbs, and intelligible pronunciation.
4. effectively explain mathematical procedures related to arithmetic, algebra, and geometry by simulating a mathematics teacher describing procedures in an oral presentation in class.

General Objective	Tasks	Skills	Language Focus	Strategies	Time Allotted
1.	Describe mathematical procedures	S	Vocabulary Numbers, negative numbers.	Reviewing and providing useful language	1 Lesson Plan

	<p>es by using proper vocabular y.</p>		<p>Verbs: add, subtract, divide by, multiply by, simplify.</p> <p>Nouns: addition, subtraction, multiplication, division, dividend, divisor, quotient, etc.</p> <p>Pronunciation</p> <p>The sound of 'tion' /suffix/</p>	<p>Heterogeneous grouping</p>	
2.	<p>Describe mathema tical procedur es by using sequenci</p>	S	<p>Vocabulary</p> <p>Squared, cubed, PI, square root</p> <p>Sequencing words: first,</p>	<p>Reviewing and providing useful language</p> <p>Heterogeneous grouping</p>	<p>1 Lesson Plan</p>

	<p>ng words and modal verbs.</p>		<p>first of all, second, third, next, then, after that, finally, lastly.</p> <p>Modal Verbs: should, have to, can, must, need.</p> <p>Pronunciation</p> <p>R controlled vowels /ɪr/</p> <p>Pronunciation of vowel sounds such as: lax u /ʊ/, diagraph /æ/, and schwa /ə/</p> <p>Modal verbs</p>		
--	--	--	--	--	--

3.	Describing various mathematical procedures by simulating a math teacher given an oral presentation.	S	<p>Vocabulary</p> <p>More mathematical terms and expressions in the areas of arithmetic, and algebra.</p> <p>Examples: parenthesis, equals, factoring, equation.</p> <p>Grammar (comparatives /superlatives)</p> <p>Examples: fewer than, less than, times as much/many as, the least common...</p>	<p>Reviewing and providing useful language</p> <p>Heterogeneous grouping</p>	1 Lesson Plan
----	---	---	---	--	---------------------

4.	Recognize vocabulary related to Geometry by listening to audios and reading excerpts from textbooks	R & L	Vocabulary Examples: acute, isosceles, obtuse, quadrilateral, scalene, angle, triangle, rectangle, square, transversal, tangent, secant, cosine parallel, measure.	Taking notes Skimming & scanning	1 Lesson Plan
----	---	-------	---	---	----------------------

Unit 3: Delivering Basic Geometry and Trigonometry Lessons

Goal: At the end of the unit, students will be able to adequately deliver basic geometry and trigonometry lessons by explaining introductory level examples and problems using the corresponding vocabulary and sequencing markers.

General Objectives: By the end of the lesson, students will be able to:

- effectively identify and use basic vocabulary and sequencing markers necessary for basic geometry lessons by completing listening and reading exercises based on authentic texts.

2. effectively identify and use basic vocabulary and sequencing markers necessary for basic trigonometry lessons by completing listening and reading exercises based on authentic texts.
3. successfully deliver basic geometry and trigonometry lessons by using the correct vocabulary, sequencing markers, and modal verbs.

General Objective	Tasks	Skills	Language Focus	Strategies	Time allotted
1	Listen to and read explanations and descriptions of elementary geometry word problems to explain basic geometry word problems to classmates.	L R S	Vocabulary Mathematical terms and expressions and sequencing markers related to geometry. Examples: first, second, next, finally, acute, equilateral, isosceles, obtuse, arc, circumference, complementary angles, cone, congruent, cylinder, diameter, etc.	Identifying key words and phrases Taking notes Checking understanding Sequencing steps	1 lesson plan
2	Listen to and read explanations	L R S	Vocabulary Mathematical terms and expressions and	Identifying key words and phrases	1 lesson plan

	and descriptions of elementary geometry word problems to explain basic trigonometry problems to classmates.		sequencing markers related to trigonometry. Examples: first, second, next, finally, cotangent, cosine, trigonometric function, secant, tangent, adjacent, etc.	Taking notes Checking understanding Sequencing steps	
3	Prepare and deliver one basic geometry lesson and one basic trigonometry lesson	L R	Vocabulary Mathematical terms and expressions in the areas of geometry and trigonometry and sequencing language. Examples: first, second, next, then, finally, acute, equilateral, isosceles, obtuse, arc, circumference, cotangent, cosine, etc.	Identifying key words and phrases Taking notes Checking understanding Sequencing steps	1 lesson plan

Conclusion

The data collected from the initial semi-structured interview and survey provided insightful information for the design of the ESP course. Even though participants' needs do not always harmonize with their particular wants, the data gathered is very useful for course designers to find commonalities and make adequate decisions. Firstly, the course was designed with a focus on speaking tasks along with vocabulary learning and pronunciation. The speaking tasks that were identified as a priority include providing instructions to students, explaining mathematical procedures, and answering students' questions. Secondly, the math content focus was on elementary and high school math. Despite the participants' interest in integrating college-level mathematics, this request is out of scope considering participants' current English proficiency and time constraints.

The results obtained from the diagnostic test have also provided valuable insight into what the focus of the future ESP course should be and a greater insight into students' current performance levels. It reaffirmed the starting point for the main goal of the course, which is as mentioned above developing oral production in tasks focused on teaching basic mathematics in English fluently and confidently. For this purpose, the input in additional listening and reading tasks, and task cycles in general, played a key role by exposing students to the vocabulary, structures, and discourse patterns that they needed. The results also showed most participants' relatively low performance in the listening and speaking tasks. In the reading tasks, the vast majority performed at an intermediate level. This ESP course thus aimed to close basic vocabulary and pronunciation gaps, in addition to covering discourse sequence markers, so that the students can start building accuracy, fluency, and confidence when explaining mathematical concepts, operations, and equations.

Chapter III. Practicum Research

The course evaluation report examines the role of differentiated instructional strategies in students' task completion in a mixed-proficiency virtual English for Specific Purposes (ESP) class. The context is a course designed for faculty members of the Mathematics Department at the Western Campus of the University of Costa Rica during the second semester of 2023. A team of three student-teachers researched, implemented, and analyzed varied strategies intended to support learners' participation and task completion according to their proficiency levels.

The course designers then gathered insights on the strategies from the perspectives of both students and supervisors during and after the course. The inclusion of perspectives from both added a comprehensive dimension to the course evaluation after its implementation, providing a well-rounded understanding of the impact of the methodology employed in the ESP course. The fact that learners will need to effectively use ESP at the workplace and for academic purposes follows from investigations into teachers' and students' beliefs that can be used to improve classroom practice (Bui, 2022). This provided a compelling rationale for the current study, and facilitated the process of gathering results to shed light on ESP teaching in a virtual setting with a variety of students at different levels of English proficiency. Bui also suggests that effective differentiation in teaching enables English as a Second Language (ESL) teachers to address various proficiency levels within the class while fostering students' development. Differentiated instruction is a method of teaching that requires teachers to comprehend and observe the similarities and differences of the students they teach (Roy et al., 2013). By acknowledging both, educators can adjust their teaching methods and strategies to meet their students' needs.

To test and confirm this statement, the course designers focused their course evaluation report on the main research question: "How do differentiated instructional

strategies impact task completion in a mixed-proficiency virtual ESP class for mathematics professors?" The course designers then examine the following supporting research questions:

1. Which strategies and techniques most effectively support students of varying proficiency levels in completing ESP tasks in a virtual environment?
2. How do students perceive the effectiveness of differentiated instruction strategies in supporting their task completion in a virtual ESP course?
3. How do supervisors evaluate the implementation and effectiveness of differentiated instruction strategies in supporting mixed-proficiency students' task completion?

These questions guided our investigation of differentiated instruction in virtual ESP teaching and informed our analysis of its impact on student engagement, learning outcomes, and overall course effectiveness.

Literature Review

This literature review examines teaching English for Specific Purposes (ESP) virtually to students with mixed levels of language proficiency. It begins by defining key concepts: English for Specific Purposes, language proficiency levels, mixed-proficiency classrooms, and virtual/online learning environments. The review then explores the main challenges faced when teaching mixed-proficiency ESP classes virtually, followed by research-based strategies and techniques to address those challenges. These strategies are organized into the following subsections: Needs Analysis and Course Design, Task-Based Language Teaching (TBLT) and Scaffolding, Technology Integration, Differentiated Instruction, Assessment, Balancing Synchronous and Asynchronous Instruction, and Cooperative Learning

Key Concepts

English for Specific Purposes (ESP) refers to teaching English to prepare learners for a specific field or occupation. ESP classes focus on developing the language skills and

vocabulary needed for a particular profession or field (Hyland, 2006). In recent years, many English as a Second Language (ESL) classes, including ESP courses, have transitioned from in-person to virtual environments due to the growth of remote or distance learning. This shift from traditional face-to-face classrooms to virtual spaces has called for a reevaluation of teaching methods, strategies, and techniques, especially when instructors are tasked with teaching mixed-proficiency learners.

Teaching a virtual ESP class with varying student language abilities presents unique challenges for educators. As Volle (2021) points out, "ESP instructors in virtual classrooms often face the daunting task of catering to diverse linguistic competencies and learning needs within a single class" (p. 32). This sentiment is echoed by Basturkmen (2022), who states, "the lack of physical proximity and limited nonverbal cues in virtual settings can make it challenging for ESP instructors to gauge individual students' comprehension levels and provide targeted support" (p. 48). Furthermore, Hyland (2019) highlights the difficulty in "facilitating authentic communicative tasks and simulations that are essential for ESP learners to develop field-specific language skills in a virtual environment" (p. 67). Overcoming these challenges requires ESP instructors to adopt innovative approaches such as utilizing interactive multimedia resources, employing differentiated instruction techniques, and leveraging virtual collaboration tools to cater to diverse proficiency levels and learning styles. Additionally, flexible approaches like offering personalized feedback, providing supplemental materials, and encouraging peer support can help address the varying needs of mixed-proficiency virtual classrooms.

To ensure an equitable learning experience for all students, it is essential that instructors employ research-based strategies and techniques tailored to mixed-level classes. As Dudley-Evans and St. John (2020) emphasize, "The methods used must accommodate diverse linguistic competencies and learning needs, requiring instructors to adopt a flexible and adaptive approach" (p. 92). This may involve incorporating

differentiated instruction, utilizing multimedia resources, and fostering peer-to-peer collaboration to support learners at various proficiency levels (Volle, 2021). By embracing these strategies, ESP instructors can create an inclusive and effective learning environment that caters to the diverse needs of their students in virtual classrooms.

In the context of ESL or ESP classes, English language proficiency refers to an individual's multifaceted ability to effectively communicate and comprehend the English language across various domains, including reading, writing, speaking, and listening. As defined by Cummins (2008), language proficiency involves "the degree to which an individual has developed competence in the target language, encompassing both conversational fluency and academic language proficiency" (p. 71). The Common European Framework of Reference for Languages (CEFR) provides a comprehensive framework for organizing language proficiency into six levels, from A1 (Basic User) to C2 (Proficient User), which can be further subdivided according to local needs (Council of Europe, 2023). At the highest levels (C1 and C2), proficient learners can understand complex texts, express themselves fluently and precisely, and use language effectively for social, academic, and professional purposes. They can comprehend implicit meanings, summarize information from various sources, and produce clear, well-structured texts on complex subjects, demonstrating controlled use of language structures and cohesive devices. However, language proficiency may vary across the four macro-skills, with learners exhibiting different levels of competence in reading, writing, listening, and speaking. Therefore, when teaching English as a Second Language or for Specific Purposes, especially in mixed-level classrooms, it is crucial to consider students' uneven proficiency levels and assess their individual strengths and weaknesses to provide targeted instruction tailored to their specific needs.

Mixed-proficiency classrooms, where students have varying levels of English language proficiency within the same course or learning group, are not uncommon in adult

ESL and ESP programs due to enrollment realities and student scheduling needs (Lightbown, 2014; Burt et al., 2008). In these heterogeneous classrooms, some students may be beginners with minimal language skills, while others may be more advanced and fluent. Although teaching language learners with a mix of proficiency levels presents challenges in any classroom environment, virtual or online classes bring additional difficulties (Thamarana, 2016). The absence of physical proximity, limited nonverbal cues, and the difficulty in facilitating authentic communicative tasks and simulations can exacerbate the challenges faced by instructors in virtual or online ESL or ESP classes with students of varying proficiency levels. Addressing these challenges requires language instructors to adopt innovative and flexible approaches tailored to the unique needs of their mixed-proficiency virtual classrooms.

One innovative approach is the strategic integration of interactive multimedia resources and virtual collaboration tools to facilitate authentic communicative tasks and simulations essential for developing field-specific language skills. These tools can provide opportunities for learners to engage in real-world scenarios, practice specialized vocabulary, and collaborate with peers, while also allowing instructors to monitor individual progress and provide targeted support. Furthermore, employing differentiated instruction techniques is a flexible approach that can cater to diverse proficiency levels and learning styles within the same virtual classroom. This may involve offering personalized feedback, providing supplemental materials at varying difficulty levels, and encouraging peer support through collaborative activities. By differentiating instruction, instructors can ensure that learners at all proficiency levels are challenged appropriately and receive the necessary scaffolding to progress in their language development. Another innovative strategy is the use of adaptive learning technologies and personalized learning paths, which can automatically adjust the content, pace, and difficulty level based on individual learners' needs and performance. This approach not only accommodates mixed proficiency levels

but also promotes learner autonomy and self-paced learning, which are particularly valuable in virtual environments. Ultimately, addressing the challenges of mixed-proficiency virtual ESP classes requires a multifaceted approach that combines innovative technological solutions, flexible instructional strategies, and a deep understanding of learners' diverse needs and backgrounds. By embracing these approaches, language instructors can create an inclusive and effective learning environment that supports learners at all proficiency levels, fostering their development of field-specific language skills and preparing them for success in their respective professional domains.

Virtual or online learning refers to educational contexts where instruction and learning occur through the Internet rather than in a physical classroom setting. As defined by Means et al. (2014), it consists of "learning experiences that are internet-mediated, with internet-based applications playing an instrumental role in both the delivery and student participation" (p. 7). Similarly, Bates (2019) describes online learning as "a form of distance education where the primary delivery mechanism is via the internet, using a variety of internet communication technologies" (p. 124). In online settings, lessons, assignments, activities, and assessments are all hosted on a digital learning platform that students access remotely through their devices. Interaction between teachers and students takes place via video conferencing, discussion boards, email, and other online communication tools. Virtual ESL and ESP classes offer several advantages. For example, they allow for increased flexibility since students can participate synchronously at scheduled class times or asynchronously based on their own schedule. Further, although online classes lack the in-person interaction of a traditional classroom, they enable personalized learning and expand access to English instruction for students regardless of their geographical location (Dung, 2020). Additionally, online ESL and ESP classes can leverage a wide range of digital resources, multimedia materials, and interactive tools to enhance language learning experiences (Blake, 2016). In essence, online ESL and ESP

provide flexible, personalized language learning opportunities through digital platforms and tools, catering to diverse learner needs and preferences. While virtual classrooms may present unique challenges, particularly in mixed-proficiency contexts, they offer a viable and accessible alternative for language learners seeking to develop their English skills in an increasingly digitalized world.

Main Challenges

Catering to learners with diverse language proficiency levels in a virtual classroom environment presents distinct challenges for instructors. To effectively differentiate instruction online, teachers must tailor materials and tasks to accommodate the varied needs of students across proficiency levels (Bielousova, 2017; Scrivener, 2012). This entails providing simplified content and activities for less proficient learners while integrating complex and challenging materials to engage advanced students. Additionally, teachers must employ varied teaching methodologies and scaffolding techniques to support beginning-level students in an online context (Scrivener, 2012; Thamarana, 2016). Unlike in a traditional classroom setting, where instructors can circulate and assist struggling students more readily, their ability to provide targeted support in virtual breakout rooms is limited. Furthermore, monitoring student progress can be challenging in an online environment due to the lack of physical proximity and the potential for technical issues or distractions to hinder effective observation. Consequently, maintaining engagement and motivation across proficiency levels during remote lessons necessitates advanced planning and preparation (Dung, 2020). Recent studies have highlighted the importance of leveraging interactive online tools and collaborative activities to sustain mixed-level student attention and participation in virtual classrooms (Bielousova, 2017; Thamarana, 2016). Techniques such as polls, chat functions, shared whiteboards, individual check-ins, peer learning, and group collaboration can effectively engage learners with varying language abilities (Scrivener, 2012; Thamarana, 2016). Thus, while teaching mixed-

proficiency ESP classes online presents unique challenges, instructors can ensure that students across proficiency levels continue developing language skills in a virtual environment by employing differentiated instruction, interactive technologies, and varied assessments.

Needs Analysis and Course Design

To enable ESP teachers to effectively differentiate and tailor instruction for mixed-proficiency classes, research emphasizes the importance of conducting a comprehensive needs analysis at the outset of an ESP course. A thorough needs analysis provides crucial insights into students' proficiency levels and learning preferences (Basturkmen, 2010; Long, 2005). Follow-up interviews and diagnostic assessments further illuminate specific needs and skills, allowing teachers to make informed decisions about course objectives, syllabus design, content selection, task sequencing, and personalized support strategies (Hyland & Hamp-Lyons, 2002). For instance, students with lower proficiency may require extra reading and listening assistance, vocabulary building, and foundational skill development before undertaking cognitively demanding, field-specific tasks (Vogt & Kantelinen, 2013). Meanwhile, students with adequate language skills but limited subject knowledge may need more scaffolding and background information to grasp subject-specific content (Basturkmen, 2010). In essence, research emphasizes that carefully analyzing needs and accurately assessing student proficiency is critical when designing ESP courses for mixed-level learners (Burt et al., 2008; Song, 2016). A combination of diagnostic tests, placement instruments, and self-assessments can enable ESP instructors to tailor their approach to accommodate diverse needs and proficiency levels. Therefore, conducting a robust needs analysis is key to understanding individual differences and preferences among learners as much as possible, thereby guiding pedagogical choices and providing differentiated instruction in mixed-proficiency ESP classes.

As can be seen, thoughtful course design is crucial to delivering effective online instruction to ESP students. Scholars (Basturkmen, 2015; Dudley-Evans & St John, 1998; Thamarana, 2016) highlight the importance of structured, well-organized courses with clear learning objectives and properly sequenced content and tasks. First, Dudley-Evans and St John (1998) emphasize that since ESP focuses on developing English skills for professional contexts, the curriculum must provide learning activities tailored to students' specific needs, fields, and goals. Without intentional design, online ESP courses risk feeling generic rather than specific and personalized. Furthermore, Basturkmen (2015) suggests that teachers should align objectives, materials, assignments, assessments, and lessons to the particular vocabulary, communication skills, and language tasks required in the target workplace or discipline. In this regard, needs analyses and collaboration with stakeholders can inform the design process by providing detailed pertinent information about those linguistic requirements. Additionally, Thamarana (2016) argues that well-structured courses also help students feel engaged and supported in the remote online environment. Organized modules, clear expectations, and identifiable sequences of lessons and assignments are especially important online. Regular interaction through discussion forums, live conferencing, and prompt feedback should be built-in as well. With a foundation of intentional, customized design, an online ESP course can deliver the profession-specific English training that diverse adult learners seek and need to advance their careers. Accordingly, thoughtful course design is key to providing impactful online ESP instruction tailored to students' needs and proficiency levels.

Task-Based Language Teaching (TBLT) and Scaffolding

One effective method for teaching mixed-proficiency ESP learners based on thoughtful course design is Task-Based Language Teaching (TBLT). As Subrahmanyam Vellanki and Bandu (2021) indicate, research suggests that TBLT fosters engagement, collaboration, and language development in online environments. TBLT centers learning

around authentic, real-world tasks, including those that ESP students will need to perform in their professional contexts, such as writing reports, giving presentations, or participating in meetings (Richards & Rodgers, 2014). By exposing learners to genuine uses of language within their field through contextualized tasks, TBLT allows them to apply specific linguistic forms in meaningful ways, regardless of proficiency level. First, in TBLT, tasks are broken down into smaller steps (Richards & Rodgers, 2014). Students at different levels can then work together to complete these steps, learning from each other in the process. For instance, Scrivener (2012) notes that teachers can provide scaffolding and support to lower-level students when completing tasks, while higher-level students can model target language for their lower-level peers. This promotes collaboration and interaction through group tasks, enabling peer feedback and mutual learning. Furthermore, Richards and Rodgers (2014) argue that the tangible outcomes motivate learners to improve their English skills. By and large, TBLT's focus on practical tasks rather than isolated language points, along with scaffolded instruction and collaboration, makes it a suitable approach for teaching mixed-proficiency ESP students.

Scaffolding instruction is a vital component of TBLT, as noted by Scrivener (2012). Scaffolding refers to the temporary support structures provided by a teacher to help students successfully complete a task that they may not be able to complete independently. In TBLT, the focus is on having students use authentic language to complete meaningful tasks. However, learners may need extra support to successfully carry out these tasks, especially in the early stages of language development. Scrivener (2012) suggests that teachers can provide this support through scaffolding techniques such as modeling, providing language frames (pre-constructed phrases or sentence structures), giving feedback, asking leading questions, and reviewing key vocabulary before the task. Additionally, Yilmaz (2011) highlights that multimedia scaffolds like videos and slide presentations are also helpful in the online environment. Other useful scaffolding

techniques include activating background knowledge through class discussions, brainstorming, or KWL (Know, Want to Know, Learned) charts (Crandall & Kaufman, 2002); using graphic organizers and charts to represent key information and relationships visually (Tomlinson, 2012); embedding comprehension checks and clarification pauses during lessons (Herrera & Murry, 2011); and using strategic pairing, where higher-proficiency students can clarify instructions and model language for lower-proficiency students (Lin & Lin, 2019). In other words, as the learner develops greater proficiency, the need for temporary support diminishes, allowing learners to complete similar tasks more independently in the future. Thus, scaffolding allows teachers to provide the right level of support needed for students to stretch their emerging language skills just beyond their current level of competence. To sum up, by systematically scaffolding instruction and materials, teachers can enable all students in a mixed-level class to engage meaningfully with content.

ESP teachers must consider their students' current proficiency levels and prior knowledge from needs analysis results when planning appropriate scaffolding for lessons and materials. As Dudley-Evans and St John (1998) explain, students have varying levels of language proficiency and carrier-content familiarity, which refers to their existing knowledge of the subject matter or professional field. Scaffolding techniques that support higher-proficiency learners may not provide enough assistance for lower-level students. Conversely, advanced students may find overly-scaffolded materials unchallenging and dull. Therefore, as Basturkmen (2015) emphasizes, teachers must first identify what students can do independently, and then determine where extra support is needed for success. For instance, lower-level students may need scaffolds such as sentence starters, visual aids, vocabulary previews, and cloze exercises to complete a task; in contrast, higher-level learners may only need occasional coaching, comprehension checks, and clarification pauses. Additionally, Herrera and Murry (2011) note that considering students'

prior knowledge is also key, as relating new information to their experiences boosts comprehension and retention. Thus, mindful scaffolding provides the right support for students to grasp new concepts and skills without over-simplifying content or boring advanced learners, while enabling meaningful engagement for all.

Technology Integration

Technology integration can also provide scaffolding to support task completion when teaching mixed-proficiency ESP learners (Ernst-Slavit & Wenger, 2006; González-Lloret, 2020; Song, 2016; Subrahmanyam Vellanki & Bandu, 2021). For instance, Ernst-Slavit and Wenger (2006) suggest that technology can be used to provide access to a wide range of resources and materials, such as online dictionaries, language learning apps, and video tutorials. These researchers recommend using multimodal instructional strategies that incorporate varied auditory, visual, hands-on, and interactive techniques. Through these means, multimodal learning engages diverse students by accommodating multiple learning styles (Subrahmanyam Vellanki & Bandu, 2021). For example, a lesson could combine direct instruction with images, videos, collaborative projects, and technology tools. This multisensory approach reinforces understanding (Subrahmanyam Vellanki & Bandu, 2021).

Furthermore, technology can be used to facilitate communication and collaboration between learners, such as through online forums and video conferencing (González-Lloret, 2020), as well as enable feedback and assessment design (Song, 2016). With teacher guidance, students can choose technologies suited to their proficiency level, promoting autonomy while ensuring full-task participation. In sum, technology integration in virtual ESP classes also offers ways to differentiate instruction in mixed-proficiency groups. Accordingly, educational technologies provide scaffolding to support diverse students in completing tasks.

Differentiated Instruction

Differentiated instruction is a pedagogical approach that acknowledges and accommodates the diverse learning needs and proficiency levels of students. As Tomlinson (2014) explains, differentiation involves modifying the content, process, and product of lessons based on students' readiness levels, interests, and learning styles. Several differentiation strategies are recommended for mixed-level classes. For example, Echevarria et al. (2017) suggest that teachers can adjust their speech and instructional materials to match proficiency levels, through techniques such as using simpler vocabulary, shorter sentences, and visuals to support beginning learners. Additionally, Scrivener (2012) notes that teachers can adapt texts and assignments to reduce linguistic complexity so that all students can work on the same content at different levels of depth and complexity. For instance, teachers can provide materials adapted to the students' levels, with simpler texts and tasks for lower-proficiency learners and more complex assignments to challenge higher-proficiency students. Furthermore, flexible grouping, for example, grouping students based on their proficiency levels, enables students to collaborate with similar-proficiency or higher-proficiency peers for targeted language practice, which involves activities and tasks tailored to their specific language needs (DeCapua & Marshall, 2015). DeCapua and Marshall (2015) also suggest that small online breakout rooms allow focused support from teachers at each level, facilitating individualized attention and guidance. While the flexibility of virtual environments can facilitate seamless differentiation, it is important to note that the teacher must actively monitor student progress, giving feedback and adjusting strategies to ensure continued growth for all proficiency levels (Echevarria et al., 2017). With the right differentiation strategies, virtual ESP classes can effectively support mixed-level task completion. All in all, differentiated instruction accommodates diverse needs in mixed-level classes through strategic content, task, and process modifications.

Formative and Summative Assessment

Another research-based recommendation in the context of mixed-level virtual classes is a balanced approach to formative and summative assessment (Echevarria et al., 2017). On the one hand, Echevarria et al. (2017) suggest that formative assessment, like observations, discussions, and practice tasks, conducted throughout several lessons, provides ongoing feedback to students and enables teachers to monitor learning and adjust instruction and scaffolding in response to individual progress and with mixed proficiency levels in mind. Formative assessment also enables teachers to gain insight into students' struggles, for example, the need of lower-proficiency students for more modeling and coaching early on, where advanced students only need clarity on task instructions (Brown & Abeywickrama, 2018). On the other hand, Brown and Abeywickrama (2018) note that summative assessment, like projects, presentations, and tests, evaluates overall learning outcomes and proficiency gains. For mixed-level classes, they recommend that rubrics with differentiated criteria can assess learners based on their proficiency level. Rubrics can also assess content, effort, and improvement, not just language accuracy, which allows lower-proficiency students to demonstrate understanding in diverse ways (Brown & Abeywickrama, 2018). Furthermore, student self-assessment can also boost awareness of growth areas (Echevarria et al., 2017). In a nutshell, balancing continuous formative feedback with summative evaluations of final work could allow teachers to calibrate instruction to proficiency levels and help ensure all virtual ESP students can successfully complete tasks.

Balancing Synchronous and Asynchronous Instruction

In addition to a balanced assessment approach for mixed-level virtual classes, research also recommends finding an optimal balance between synchronous and asynchronous instruction for ESP classes (Cunningham, 2014; Huang, 2015; Ozturk, 2015; Satar & Özdener, 2008). Cunningham (2014) notes that synchronous instruction,

like live video lessons, provides opportunities for real-time teacher-student and student-student interaction, creating a sense of community and allowing for modeling, immediate feedback, and clarification. This benefits lower-level students, who rely more on teacher and peer input and guidance. However, it may negatively affect higher-level students' engagement if instruction is not purposefully differentiated (Huang, 2015). In contrast, asynchronous instruction, with resources like discussion boards and prerecorded video lessons, allows students to work at their own pace and revisit material as needed (Ozturk, 2015). An important drawback is that students lose the benefits of live interaction. To balance the drawbacks of each modality, Satar and Özdener (2008) suggest blending synchronous group work, lectures, and discussions with self-paced asynchronous assignments and resources catering to different learning styles and proficiency needs. The synchronous components promote engagement and collaboration while the asynchronous components provide flexibility and support. Research studies have found that this balanced multimodal approach can potentially enhance diverse students' linguistic development, content knowledge acquisition, and overall satisfaction in mixed-level ESP courses (Cunningham, 2014; Satar & Özdener, 2008). Hence, an optimal balance of synchronous and asynchronous instruction could accommodate varying proficiency levels in mixed-proficiency ESP classes.

Cooperative Learning

Another basic principle for teaching mixed-level classes is cooperative learning. As Echevarria et al. (2017) explain, cooperative learning, where students work in small groups to complete tasks and projects, is commonly recommended for linguistically diverse classes. Key features of cooperative learning include positive interdependence, where group members rely on each other to accomplish shared goals; individual accountability, where each student is responsible for their own learning; heterogeneous grouping, where groups are formed with students of varying proficiency levels; and structured student

interaction, where students are provided with guidelines for effective collaboration (Slavin, 1995). A key principle of the approach is that when students of varying proficiency levels work together in small groups, they can support and learn from one another through peer modeling, scaffolding, and feedback (Johnson & Johnson, 1999). Teachers can structure cooperative activities so that students at different English proficiency levels interact and learn from one another across proficiency levels. For instance, Slavin (1995) notes that higher-level students can provide examples of accurate language use and comprehension strategies, while lower-level students can gain confidence and motivation through peer support.

Nevertheless, Ortiz and Fránquiz (2017) suggest that homogeneous grouping also has value. Teachers may group students homogeneously by proficiency level for specific tasks, enabling the teacher to provide lower-level students with more scaffolded instruction, while challenging advanced learners with less guided tasks. In either configuration, group tasks and projects create opportunities for meaningful interaction and language practice, as negotiating meaning through authentic dialogue and collaboration promotes language development irrespective of proficiency level (Lee & VanPatten, 2003). This may facilitate a sense of community in a virtual environment where lower-level and higher-level learners support each other's linguistic development. Fostering teamwork and communication is ideal training for real-world professional contexts where collaboration is key. Thus, cooperative learning is ideal for mixed-level ESP classes while aligning with the goals of preparing students for authentic professional communication and collaboration.

This literature review has synthesized principles, strategies, and techniques that teachers can utilize to help diverse students overcome challenges and effectively accomplish tasks in online ESP courses, an increasingly important domain of practice. Key research-supported instructional strategies for successfully teaching mixed-proficiency ESP students virtually include differentiating and scaffolding instruction, conducting

thorough needs analyzes to inform course design, planning collaborative tasks, balancing formative and summative assessment, and following a balanced multimodal approach to virtual instruction. Task-based instruction and technology integration are also core principles and methods for teaching mixed-proficiency ESP classes. Teachers can draw upon a variety of these strategies and techniques to accommodate all learners regardless of their proficiency level. The literature highlights that a learner-centered approach, guided by ongoing needs assessment and formative feedback, is critical to ensuring equitable access and achievement for all students in mixed-ability virtual ESP classes. However, further research is still needed in the Costa Rican context on the effectiveness of these strategies across disciplines, age groups, and platforms. Overall, this review aims to somewhat fill the gap in research by exploring instructional approaches for teaching mixed-proficiency ESP classes virtually in the Costa Rican context.

Methodology

Research Approach

The course designers followed a qualitative research approach to look into and answer the research questions, specifically, assessing the impact of mixed-level strategies of task completion and the perceptions of students and supervisors of the implementation of the strategies. Qualitative research is characterized by an interpretative paradigm, which emphasizes subjective experiences and the meanings they have for an individual. Therefore, the subjective views of a researcher on a particular situation play a vital part in the study results (Starman, 2013). This approach enables a deeper understanding of individuals' perspectives within a specific context. To analyze the impact of strategies used to assist students in the mixed-proficiency virtual ESP course based on students', supervisors', and their own perceptions, the student-teachers collected data from a series of instruments: two surveys for students, an unstructured interview for supervisors, unstructured observations by the student-teachers, and a diagnostic test and a final

speaking test for students. By yielding a variety of qualitative data, these instruments provided insights to assess the usefulness of differentiation strategies based on detailed descriptions from those directly involved in the virtual ESP course. The qualitative findings will inform practical implications for differentiating instruction in similar virtual ESP contexts.

Context

This study was carried out in an ESP course for faculty members of the Mathematics Department at the Western Campus of the University of Costa Rica. The course is part of a curriculum-design project aiming to develop a new bachelor's degree program that will prepare bilingual mathematics educators. This program is in the early stages, with current efforts focused on enabling mathematics professors to teach their specialized content courses in English. Equipping faculty with ESP skills and bilingual teaching competencies will lay the foundation for the planned degree program that will train dual-language mathematics teachers. This research analyzes the initial ESP course offering for mathematics faculty, providing key insights, particularly into teaching a mixed-proficiency class, that will shed light on the ongoing development of the curriculum and the larger bilingual teacher preparation initiative.

The learner population in the ESP course consisted of 16 mathematics professors from the bachelor's degree program in mathematics education. Of these, 12 participants completed the course and made up the sample of the study. The participants specialized in various mathematics subfields, including theoretical mathematics, applied mathematics, and mathematics education. Their ages ranged from 35 to 55 years old, and they were distributed equally into 6 women and 6 men. All of them had comparable prior experience learning English acquired from their high school education or private lessons. In a diagnostic language test in the needs analysis at the outset, most participants' performance in the tasks could be classified as that of basic English users comparable to

an A1 based on the *Qualitative Aspects of Spoken Language Use* of the CEFR, with listening and speaking as the main areas of difficulty. Following the same descriptors, only three students could be classified as intermediate, with a performance comparable to B1. During the needs analysis process and prior to the start of the course, the group reported a high degree of motivation to develop their linguistic skills to teach mathematics effectively in English. Their dedication to improving their ESP proficiency was key to their success in the course.

Instruments

Assessment Survey for Self-Reflection and Course Improvement. The first instrument was a self-assessment survey administered to students in the ESP course. The survey was designed by the student-teachers and included Likert-scale statements about class preparedness, participation, and engagement. It also contained three open-ended questions asking students to share their opinions on the course overall, areas needing improvement, and their favorite activities (see Appendix E). This survey provided valuable insights into the learner population and their experiences as key stakeholders. Their feedback helped evaluate course design and success, and obtain data for future modifications precisely when it came to the students' feelings when working with classmates with the same or different English proficiency levels and their opinions on differentiated worksheets according to their specific English language proficiency. The answers provided by the participants in the aforementioned areas directly addressed the research questions posed for this project. To easily distribute and compile survey responses, the course designers constructed the questionnaire using Google Forms. Students completed the survey through this online platform in about 15 minutes, and it was administered only once during the course. The combination of closed- and open-ended items yielded comprehensive qualitative data to understand students' perspectives on the ESP course and notably some of their views on the strategies used in a mixed-level class.

Mixed-levels Strategy Survey. The second instrument was a mixed-level of proficiency strategy survey with six open-ended questions (see Appendix F). This survey was also designed by the student-teachers and asked students to evaluate the effectiveness of same-level versus mixed-level teamwork. It also inquired about the benefits of individualized materials tailored to each learner's proficiency. The instrument was based on the process of validity that involves accumulating relevant evidence to provide a sound scientific basis for the proposed score interpretations. It is the interpretations of test scores for proposed uses that are evaluated, not the test itself (Standards for Educational and Psychological Testing, 2014). Administering this survey allowed the researchers to gather students' impressions about the usefulness of learning with the differentiated instruction strategies drawn from related literature and implemented in the course. These strategies aimed to facilitate the completion of ESP tasks for the mixed-level class. Like the first survey, the course designers created this questionnaire on Google Forms for easy distribution and response compilation. Students completed this survey in approximately 20 minutes. The survey was administered during the final class meeting to capture learners' perspectives after experiencing the strategies throughout the semester. The open-ended responses offered insights into the perceived value of mixed-levels approaches from the students' point of view.

Unstructured Observation. The third instrument involved unstructured observations and analysis of the mixed-level of proficiency strategies implemented in the course. Observation in qualitative research is one of the oldest and most fundamental research methods approaches. Observation involves collecting data using one's senses, especially looking and listening in a systematic and meaningful way (McKechnie, 2008). To clarify, this process required the researchers to observe and analyze how the different strategies were applied without strict guidelines, paying close attention to details and emerging patterns to gain valuable insights. Drawing on the literature reviewed, the course

designers observed and took notes on the strategies in action over the semester. They recorded the lessons and documented observations related to the research focus. The use of video-recording allowed the researchers to play the video over and over, aiding the data analysis process and reducing personal observational bias (Lopez & Whitehead, 2012). This method can also be effectively used for intervention studies to compare pre- and post-change in practice. Comparing real-time observations to the literature allowed the instructors to assess the effectiveness and limitations of the selected mixed-levels approach, as well as draw recommendations for upcoming courses. Their in-class notes also helped determine whether students could competently complete tasks using the differentiated strategies or not. This instrument provided direct evidence to complement the survey and interview data. The instructors' firsthand observations throughout the semester offered valuable insights into how the strategies functioned in the authentic context of the virtual ESP classroom.

Semi-structured Interview. Another key instrument was a semi-structured interview with one of the practicum program supervisors (see Appendix G). The analysis of the data from the surveys and the semi-structured interview, provided a repertoire of possibilities. It is sufficiently structured to address specific topics related to a phenomenon of study while leaving space for participants to offer new meanings to the focus of the study (Galletta, 2013). This instrument allowed the course designers to gain valuable feedback from a field expert who has been involved in ESP course design and delivery at the University of Costa Rica for more than five years. The supervisor's insights and knowledge of teaching English for specific Purposes expanded opportunities for reflection and analysis on the research focus. Her outside perspective provided important considerations. The semi-structured interview was conducted during a feedback session with the instructor team and lasted approximately 30 minutes. To capture the data, the semi-structured interview was recorded and transcribed (key phrases) for ongoing

reference during the study. This interview supplemented the instructors' viewpoints and offered an additional layer of expert input to inform the research.

Diagnostic Test. A diagnostic test was administered in the second stage of the initial needs analysis leading to the design of the course. The test assessed listening, speaking, and reading skills. Writing was excluded based on previous needs analysis data indicating that listening and speaking were top priorities for the ESP course. While reading development was not a primary goal, two reading tasks were included since a variety of texts would provide language input in the course. The diagnostic test established baseline proficiency in ESP tasks similar to those that would be taught in the course across the macro skills. A noteworthy aspect to highlight is that, as part of the study and analysis of this instrument, only the results obtained in the speaking section of the diagnostic test were considered and compared to a final speaking test given at the end of the course. These results contributed to directly addressing the research questions posed.

Final Speaking Test. The final instrument was a speaking test administered at the end of the course. This test presented mathematical operations related to algebra, geometry, and trigonometry (see Appendix H). Students had to read these problems aloud, applying the key vocabulary and expressions taught during the semester. The test evaluated students' speaking performance and measured language proficiency gains thanks to the implementation of strategies such as scaffolding and all its variables during the ESP course. It required students to demonstrate expertise in pronouncing mathematical terms and concepts in English from the basic to the most complex with the assistance of visual supports and differentiated approaches based on the participants' level of proficiency. Results from this speaking test provided qualitative data on students' oral fluency development. Along with measuring individual progress, the scores were analyzed to assess overall gains. The speaking test offered direct evidence of students' improved linguistic skills for communicating mathematical knowledge in English. These

data were crucial for both evaluating the success of the course and for supporting conclusions drawn from the other instruments about the usefulness of the mixed-levels strategies.

Procedures

Data Collection. The first step to start collecting data was coordinating among the course designers the creation of the instruments and respective protocols based on relevant literature, course evaluation focus, and practicum requirements. Next, the course designers created a schedule to watch, analyze, and discuss class recordings and findings from the various instruments. After that, the course designers set specific dates for administering the surveys and conducting the semi-structured interviews. To manage the diagnostic and final speaking assessments efficiently within the time constraints, the course designers evenly divided the students to evaluate them individually and to later discuss their performance. Finally, after the course designers coordinated the administration of each instrument, they proceeded with the subsequent analysis of the collected data to answer the research questions proposed. Establishing organized protocols and timelines enabled the effective collection of qualitative insights through the surveys, semi-structured interview, and class observations as well as qualitative performance data from the pre- and post-tests. These coordinated efforts of the course designers allowed for a comprehensive qualitative research approach ensuring a throughout examination of the strategies and outcomes.

Data Analysis. Effective data analysis procedures are essential for meaningful insights and conclusions to the research. The student-teachers devoted time to carefully read, analyze, and interpret the participants' responses in the Assessment Survey for Self-Reflection and Course Improvement, the Semi-Structured Interview, and the Mixed-Levels Strategy Input Survey. These responses were organized into tables for further comparison and examination among the respondents.

Regarding the data obtained through the Unstructured Observations, the student-teachers made a compilation of the notes taken in class during the semester and observed the Zoom recordings to corroborate how the strategies seemed to influence the learners' performance. Following that, the data gathered as a result of the Diagnostic Test and the Final Speaking Test was used for comparison purposes in a table that revealed how the learners progressed in their language learning process over time.

To ensure a structured approach to the analysis of the data from the surveys and the semi-structured interview, the student-teachers employed thematic analysis as described by Braun and Clarke (2006). This process involved multiple steps, beginning with data familiarization. The student-teachers carefully read the responses to gain a comprehensive understanding of participants' perspectives. Once familiarized with the data, they proceeded with initial coding by highlighting key phrases, words, and ideas that frequently appeared in the different instruments. These preliminary codes were generated based on recurring themes, significant observations, and prior research on differentiated instruction in mixed-proficiency classrooms.

After coding, the student-teachers engaged in theme identification. The codes were systematically grouped into broader categories, ensuring alignment with the research questions. The primary themes that emerged included Formative and Summative Assessment, Synchronous and Asynchronous Instruction, Task-Based Learning Teaching and Scaffolding, Technology Integration, Cooperative Learning, and Differentiated Instruction. Each theme was broken down into subcategories to provide a more granular analysis. For example, within the Cooperative Learning category, subcategories such as mixed-ability grouping and same-ability grouping were established.

To ensure reliability, the student-teachers conducted peer debriefing sessions where they reviewed and discussed the identified categories and subcategories.

Discrepancies were resolved through discussion and reference to existing literature, enhancing the consistency and credibility of the analysis.

In addition to the thematic categorization of survey and interview responses, the student-teachers analyzed qualitative observations collected from unstructured classroom observations and Zoom recordings. Through a thematic analysis similar to that used for those instruments, these sources were used to corroborate the identified themes and assess how the applied strategies seemed to influence student performance and engagement over time.

Finally, a comparative analysis was conducted using data from the Diagnostic Test and the Final Speaking Test. Performance data was organized into tables to track individual progress and overall class improvement. This step allowed the student-teachers to identify trends and of the implemented instructional strategies.

By following this thematic analysis approach, the student-teachers ensured that their findings were well-supported and aligned with the study's objectives.

Results

The following section presents selected data sets with the results obtained after applying some of the strategies that the literature suggests ESL teachers can use to assist students in completing tasks in a mixed-level classroom. The results are organized by strategy, and each strategy includes a brief description of what was done during the course. Additionally, it includes valuable insights reported by one of the supervisors through a semi-structured interview and by the student-teachers through unstructured observations. Furthermore, a comparison and contrast of data collected from the diagnostic test and post-test are included. This section aims to provide a comprehensive analysis of the strategies' impact in a mixed-level classroom setting. Using thematic analysis as the analytical method, the results highlight key patterns and themes that emerged from the data.

Formative Assessments

Formative assessments such as observations, discussions, and practice tasks were part of the strategies implemented to assist students in a mixed-level virtual class. Table 9 shows how the main tasks every week were very similar to the final test and term oral project that students had to complete at the end of the course. In the final test, students had to read mathematical operations and equations aloud by using correct vocabulary, sequencing language, and context-specific structures. In the term oral project, students had to prepare and record a ten-minute teaching demonstration in which they explained how to solve a series of basic mathematical operations and equations provided by the instructors. Therefore, the main tasks shown in Table 9 can be considered as practice tasks that prepared students to be successful in the summative assessments. The results in Table 18, comparing the students' performance in the Diagnostic Test and the Final Speaking Test, suggest that the consistent practice on this type of task and the ongoing feedback provided by instructors in the breakout rooms and in the main session throughout the course might have contributed to task completion in the summative assessments.

Table 9

Main Task Description per Week (practice tasks)

Week	Main Task
W2	Role-play a scenario between a math teacher and a student explaining algebra basics.
W3	Role-play a scenario between two students explaining the order of operations.
W4	Role-play a scenario about explaining a geometry concept.

- W5 Role-play a scenario between a teacher and 2 students about calculating areas and reading equations.
- W7 Explain a series of steps to solve a quadratic equation.
- W8 Construct an explanation of two-step equation, simulating a mathematics teacher preparing for a class about basic algebra.
- W10 Construct a conversation related to geometry between a mathematics teacher and a student.
- W11 Construct a simple but clear explanation of Pythagorean theorem, simulating a mathematics teacher preparing for a class about basic trigonometry.
- W12 Construct a simple, but clear explanation of how to find a missing side in a triangle, simulating a mathematics teacher preparing for a class about basic trigonometry.
-

Self-Assessment. A self-assessment was administered during the course via the survey for self-reflection and course improvement. Table 10 provides a summary of the students' responses from the survey regarding their perceptions about their class preparedness, participation, and engagement during the course.

Table 10

Students' Perceptions of their Preparedness, Participation, and Engagement in the Course

Self-Evaluation Statements

Usually

Sometimes

Hardly
Ever

I am ready to start the class on time.	S3, S4, S5, S7, S9, S10	S11, S12	S2, S8
I come to class prepared. I do all course tasks and activities assigned as homework.	S2, S3, S4, S5, S7, S9, S10, S12	S8, S11	
I have my camera on during class.	S3, S4, S5, S10, S11, S12	S2, S7	S8, S9
I participate and make contributions to class and group activities.	S2, S3, S4, S5, S8, S10, S11	S7, S9, S12	
I volunteer to answer the teacher's questions.	S3, S4, S5, S10	S2, S9, S11, S12	S7, S8

How would you rate your overall class preparedness, participation, and engagement?

Deficient	1	2	3	4	5	Outstanding
		S8	S3, S7	S4, S5, S10, S11, S12	S2, S9	

As can be seen, most of the students reported that they were usually ready to start class on time and came prepared with all of their homework done. In terms of participation and engagement, most students rated themselves as usually having their cameras on and participating actively in class. The aspect where students showed the least participation was when the teachers asked for volunteers. A possible explanation is addressed in

Tables 13 and 14 by a student who stated her preference to work in small groups because speaking in a big group embarrassed her. Additionally, another student suggested the implementation of a different strategy to ask for participation due to the lack of students volunteering to participate. As can be seen, students' oral participation and engagement before the whole class seems to be one of the biggest remaining challenges in online learning nowadays. The other, more positive results suggest that students' preparedness, engagement, and participation might have contributed to achieve task completion. Therefore, having students complete a self-assessment might have had a positive effect by making them more self-aware of the effort they were bringing to class and by being more intentional about their participation.

TBLT and Balancing Synchronous and Asynchronous Instruction

TBLT and balancing synchronous and asynchronous instruction is the approach expected to be used by the course design team as part of a formal requirement of the Master's Program in English Teaching as a Foreign Language. Thematic analysis revealed that TBLT, along with synchronous and asynchronous instruction, emerged as a primary theme in the data. Based on the analysis, these strategies played a crucial role in supporting students' task completion within a mixed-level class. Tables 11 and 12 show students' opinions about the course and what they liked the most about it. These opinions were obtained via open-ended questions from the assessment survey for self-reflection and course improvement. As shown, students seemed to appreciate the course design the most, more specifically the way it was tailored to focus on mathematics education. Students highlighted the proper planning, organization, and preparation of the instructors as well as the variety of activities. In Table 11, S2 described the course and activities as well planned, engaging, practical, and conversational. In addition to that, in Table 12, S10 stated that the most enjoyable part of the course was the well-planned activities, the flawless organization, and the student-teachers' enthusiasm to teach. The answers

suggest that the mixed-levels strategies of TBLT and balancing synchronous and asynchronous instruction may have contributed to the positive perception students had of the course. In Table 11, S9 stated that it was a good course to practice writing, pronunciation, speaking and listening in English. Language skills were purposely distributed across the synchronous and asynchronous session creating an optimal balance. Synchronous instruction targeted foremost speaking skills which fostered engagement and collaboration, while asynchronous instruction was an opportunity for students to dive deeper into micro skills of the language such as grammar and pronunciation. Once again, the answers suggest students' overall satisfaction with the course organization.

Table 11

Students' Opinions about the Course

What do you think about this course?

Students	Students' responses
S1	No response.
S2	I think it was well planned, the activities were engaging, time goes by fast, and the best part is that it is very practical and conversational.
S3	The course is good and it's suitable, but I could not dedicate enough time.
S4	I am enjoying this class.
S5	The course in general has exceeded my expectations. I have learned something new in every class such as pronunciation and how to structure explanations of mathematical procedures. As a matter of fact, I am enjoying this course a lot. I

must admit that I would like to have more time to go over what I have learned in class. However, I barely have time to complete homework. I appreciate the effort on preparing every single class.

- S6 No response
- S7 The response from the instructors to help and answer questions has been excellent, as well as the activities.
- S8 I think that not all the people are at the same level, so it makes it difficult for some of them. It could have been more beneficial to others to split the group. They could have advanced faster because of the knowledge they already had on the language.
- S9 It is a good course to practice writing, pronunciation, speaking and listening in English.
- S10 I think it is a great course due to its focus on math. It is hard to find that somewhere else.
- S11 It is noticeable how well prepared the instructors are. In fact, that is part of my motivation. The classes are prepared with responsibility, they are engaging, and we learn a lot.
- S12 I like the class dynamics.
-

Table 12*Students' Opinions about their Most Enjoyable Part of the Course*

What do you like or enjoy most about this course?

Students	Students' responses
S1	No response
S2	I think the type of activities were simple, but I learned a lot.
S3	I love learning anything, and English is very important. I enjoy improving my listening and pronunciation.
S4	All your efforts to prepare the class using different techniques.
S5	I agree with the methodology they have implemented. It is interesting how they have adapted second language learning methodologies to the math content. I think we can learn a lot from this. What I like the most is when they help us pronounce the words.
S6	No response
S7	I like the variety of activities a lot. I enjoy it most when working in groups, maybe because I am embarrassed to participate.
S8	The type of activities was varied.
S9	Everything has been fine so far.

- S10 I really like how they prepare the classes. They are always very thoughtful with well planned activities, flawless organization, and enthusiasm to teach.
- S11 I like the design of the course with its focus on math.
- S12 You have created a safe environment to participate and learn.
-

Technology Integration

As mentioned above, technology integration may have contributed to the positive perception students had about the course. The theme of technology integration also emerged from the thematic analysis, as evidenced by repeated references from the students to the variety and thoughtful planning of activities, as well as to the overall enjoyment students experienced in the class. The course took place in a virtual setting, and therefore, many of the tasks incorporated the use of technology tools and platforms such as Zoom, YouTube, Power Point Presentations, Kahoot, Quizizz, Padlet, Mentimeter, Google Classroom, Drive, Jamboard, Educima (online word search), Interacty (online memory game), Wordwall, Oxford Learner's Dictionaries, and other online dictionaries. The student-teachers tried to vary the integration of these technology tools and platforms in every lesson so that students would have a diversity of task types in every class.

According to unstructured observations, in Week 10, when students practiced geometry-related vocabulary by completing a set of sentences using the platform Mentimeter in the warm-up, all students appeared to be engaged in the task and were able to complete it. In Week 4, students used Jamboard during the post-task. By taking turns, student A described some geometric shapes, lines, points, and planes using the vocabulary and definitions learned previously, and student B drew the figure based on the description. The task was simple yet multisensory since it involved listening to the

description, drawing, and seeing the visual of the drawing. Students seemed to be actively engaged in the task and working with enthusiasm. Laughter and excitement were observed as they enjoyed the task. The students' positive energy reflected their involvement and enjoyment of the task. This was further supported by their comments, as found in Table 12, from the assessment survey for self-reflection and course improvement. S7 and S8 highlighted that they liked the variety of activities very much. With these statements, students were likely referring, among other features of the activities, to the way instructors integrated technology and multimodal instructional strategies in every lesson. S4 and S5 also mentioned that they enjoyed the course a lot. In addition to that, S10 stated "I really like how they prepare the classes. They are always very thoughtful with well planned activities, flawless organization, and enthusiasm to teach". The answers suggest that the activities they engaged in class overall could have contributed to their perception of the course as enjoyable, including those carried out through diverse technology tools.

TBLT Scaffolding Instruction Principle

Scaffolding instruction emerged as a key theme in the data, reflecting its role as a core principle of TBLT that supported task completion throughout the course. One of the main goals of the course was to get students to explain mathematical procedures using appropriate mathematics vocabulary, grammatical structure, and pronunciation. Table 18 shows that S7, S8, and S11 were not able to achieve task completion in the diagnostic test. However, in the final speaking test, the same students were able to complete the task successfully. The result suggests that the scaffolding instruction used by the instructors throughout the course may have helped the learners complete the same type of task at the end of the course.

The course design team planned each unit around scaffolding instruction. Unit 1 was focused on delivering arithmetic, algebra, and geometry input along with semi-

structured output tasks. For example, in Unit 1, Week 2, Handout 3, the team planned a semi-structured role-play as part of the main task. Based on unstructured observations, students were able to complete the task successfully. From there, the team scaffolded to Unit 2, Delivering Arithmetic, Algebra and Geometry Lessons, and Unit 3, Delivering Basic Geometry and Trigonometry Lessons, mixing semi-structured output tasks with less structured tasks for higher-level students. For example, in Unit 3, Week 11, students had to build an explanation about the Pythagorean Theorem. For this task, the team created two versions of the handout. One version was for lower-level learners and the other one was for higher-level learners. Higher-level students were observed fluently building their explanations with minimal assistance, demonstrating strong language skills and confidence in their responses. In contrast, the lower-level students relied heavily on the guide to construct their explanations but found the task challenging. The task was only partially completed due to time constraints, with lower-level students needing significantly more time to fully engage and complete the task. In Unit 3, Week 12, students had to build an explanation about finding the missing side of a triangle. The task was scaffolded for both higher-level and lower-level students. Higher-level students had to watch a video that explained the procedure and had it as a reference to build their own explanation. They were observed actively engaged in building and practicing the explanation. They applied the vocabulary previously learned in the pre-tasks. In addition to working through the mathematics explanation, students frequently asked specific questions about the pronunciation of challenging vocabulary words. Their focus on pronunciation indicated both an awareness of language learning and a desire to use the terms accurately while completing the task. On the other hand, lower-level students watched the same video and completed a pre-made script. The pre-made script seemed to be highly beneficial for lower-level students as it allowed them more time to practice both the content and pronunciation. With the script, they could rehearse the language orally, focusing on the

pronunciation of challenging words without the added pressure of constructing their own sentences. This extra practice helped build their confidence and ensured they could engage more effectively with the task. Additionally, the student-teachers were actively providing immediate feedback, which further supported the students' progress by correcting errors and reinforcing proper pronunciation.

Finally, it is relevant to highlight that the team also scaffolded the mathematics content from basic content such as Arithmetic in Unit 1 to more advanced mathematics content such as Trigonometry in Unit 3. This gradual scaffolding was implemented with the intent of allowing students to have a strong grasp of simpler concepts before tackling more challenging mathematical language.

Cooperative Learning

Cooperative learning emerged as a primary theme in the data, highlighting its role as a mixed-level strategy that fostered collaboration and peer support. Throughout the course, both heterogeneous and homogeneous grouping techniques were implemented to enhance student engagement in group tasks. In Table 11, S8 shows a debatable perception on the aforementioned strategy. S8 believes that since not all the people were at the same level, it could have been more beneficial to others to split the group. This way, higher-level students could have advanced faster because of the knowledge they already had of the language. S8 suggests that homogenous grouping might be more beneficial for an ESL mixed-level course. In Table 13, which provides valuable insights on how to improve the course, other students' answers to another question in the assessment survey for self-reflection and course improvement offer contrasting viewpoints.

Table 13*Students' Opinions about Course Improvements*

How can we improve this course?	
Students	Students' responses
S1	No response.
S2	It is hard to say, I found it very enriching.
S3	In my case, I need to improve listening and speaking. More practice in this sense.
S4	You are doing great!
S5	Perhaps one suggestion would be to find another strategy to ask for participation. I understand it can be complicated due to the virtual setting. However, I think people are here because they want to learn. I feel like sometimes we have let you down when you ask for volunteers to participate. I sometimes do not volunteer myself to give the opportunity to somebody else.
S6	No response
S7	I would like to have access to the recording of the class to go over the activities. I think it will be useful to go over pronunciation and other important aspects of the class.
S8	Since there are several instructors, the groups could have been split by proficiency level.

- S9 Perhaps, when you apply listening quizzes make sure the volume is adequate, even if you repeat the audio twice. On a couple of occasions, I have not perceived a good audio volume.
- S10 It is hard to find something to improve the course. Perhaps the course could progress faster with the intention of covering more content by the end of the semester.
- S11 Everything is good.
- S12 I do not have any suggestions.
-

As shown, half of the students seemed satisfied considering they did not provide any suggestions or did not respond at all. This suggests that the selected strategies may have been satisfactory for those students. The others provided different suggestions, most of them unrelated among them, but definitely valid enough to consider for a future course. As can be seen, S8 did provide an answer directly related to mixed-levels strategies like in the question addressed in Table 11, specifically calling for like-ability grouping. In this regard, the request can be fulfilled in some cases, but arguably not in all. Most ESP courses likely have mixed-ability students. It would not be viable, or even reasonable, to have one instructor teaching 2 advanced students, another one teaching 10 basic students, and one more teaching 4 intermediate students. However, it is feasible to group students by proficiency level to complete certain tasks, which in fact was one of the strategies implemented during the course. For example, in Unit 3, Week 11 and Week 12, students worked as a whole group during the warm-up and pre-tasks. After that, students were split into like-ability groups during the main-task to differentiate instruction. Finally, students were back to work as a whole group to wrap up the class with a post-task.

Moving forward, it is important to explore how students perceive working in like-ability versus mixed-ability groups. Understanding their experiences in these different group settings can provide valuable insights into their engagement, confidence, and collaboration. The next set of data will address students' perspective on how grouping strategies influenced their learning and overall comfort. Table 14 presents students' feelings when working with classmates who have the same or similar English proficiency.

Table 14

Students' Feelings when Working with Classmates with the Same or Similar English Proficiency Level

Students	Students' Opinions
S1	Good, I liked it.
S2	Good, I think we were always in a safe environment.
S3	It was very good, together we corrected each other's mistakes.
S4	Very good, we supported each other when working together.
S5	Very comfortable. I consider the strategy to be appropriate.
S6	Good, I have really liked the course.
S7	It was very enriching, we completed the work as a team. If we did not know, we tried to find the accurate vocabulary.
S8	Not as frustrated.
S9	Comfortable.
S10	Very good, it was a collective learning that I consider very valuable. Being all under similar circumstances made it feel less overwhelming when learning.
S11	Good, but several times we had to work with classmates with a higher level than mine.
S12	Good, I liked it. Good, it makes it a little more comfortable to try pronunciation (less embarrassing).

Based on the data, most of the students felt good or comfortable when working with classmates who had the same or similar English language proficiency. Another notable result is that none of the students reported a negative perception about this strategy. As can be seen, with this strategy most students felt that they were in a safe and supporting environment as well as less overwhelmed, frustrated, or embarrassed. Thus, course design must contemplate same-level grouping, if not always, at least consistently to

promote a safe atmosphere and lower the affective filter. Building on these findings, it is also important to examine how students feel when working with classmates of varying proficiency levels, as shown in Table 15.

Table 15

Students' Feelings when Working with Classmates with a different English Proficiency

Level

Students	Students' Opinions
S1	It is challenging, but I liked it.
S2	Good as well, it gave us the opportunity to learn from each other.
S3	Good as well since it is about giving and receiving help from each other.
S4	I liked it because I could support the ones with a lower level. I also learned from the ones who had a higher level.
S5	I feel like the level of trust that we have as a group allows every session, with any person, to feel good.
S6	Good.
S7	With classmates who had a higher level, I felt intimidated at first. However, after that, I gave myself the chance to try and learn from them.
S8	Stressed since I did not have the same English proficiency.
S9	A little strange, but bearable.
S10	With classmates who had a higher level, I felt comfortable enough to ask questions with the intention of improving. With classmates who had a lower level than mine, I tried my best to help them with their mistakes.
S11	Good, they help a lot.
S12	It was also good because I could ask them questions with trust. I always received support from my classmates.

As can be seen, most students also felt good when working with classmates who had a different English level proficiency. S4 mentions, "I like it because I can support the ones with a lower level. I also learned from the ones who had a higher level". This answer suggests that the student sees the value in one of the strategies implemented in the class, which was cooperative learning. S2 perceives the strategy as a way to learn from each other and S3 sees it as a way of giving and receiving help from each other. These answers suggest that the mixed-levels strategies were perceived to have a beneficial effect.

Another notable result is that just 4 out of 12 students reported feeling comfortable enough to manage working with more proficient students. However, they perceived it to be

strange, challenging, intimidating, and stressful. It is worthwhile to note that three of these comments come from students whose language proficiency is at a basic level. Therefore, it may be concluded that beginner language learners are quite expected to have those feelings in a mixed-level class. S7 stated that “with classmates who had a higher level, I felt intimidated at first. However, after a while, I gave myself the chance to try and learn from them.” S7 had a basic English proficiency level and was aware of the fact that there were classmates that were more proficient in the target language than he was. He recognized the threat and felt intimidated. Nevertheless, after some time, his feelings seemed to have changed to the point that he was feeling comfortable enough to give himself the chance to try and learn from them. His classmates were not a threat anymore, but a learning resource. The answer suggests that the mixed-level strategies used by the instructors helped S7 transition from a higher affective filter to a lower and more balanced affective filter. Similarly, S5 indicated that the level of trust that they had as a group allowed every session, with any person, to feel good. The answer could imply that building a classroom community that embraces errors as part of the students’ unique language learning process might ensure the successful implementation of the cooperative learning strategy and reduce the affective filter. Most importantly, reducing the affective filter may foster students’ engagement in task completion. Expanding on these findings, Table 16 shows students’ comments on their preference between working with classmates who have a similar or different English language proficiency.

Table 16

Preference between Working with Classmates who Have a Similar or Different English Proficiency

Students	English Proficiency	Students’ Opinions
S1	Basic	Similar because we help each other. However, classmates with a higher level help us improve.

S2	Basic	I believe that when the proficiency level is similar, it helps with confidence.
S3	Basic	Similar because learning and feedback occur in both directions.
S4	Intermediate	I prefer to work with peers who have a similar proficiency level. I have learned from all of them anyway.
S5	Intermediate	I think both are beneficial. Either because I could learn something new or because I could teach someone something I already know.
S6	Basic	I consider that it works well in both cases. For example, in the case of higher-level students, my classmates have had a lot of patience to teach me.
S7	Basic	In my case, similar, because producing is more accessible for me and it allowed me to contribute more.
S8	Basic	In some way, being with those who had a higher proficiency level would help with pronunciation or explanations of something. However, at the same time I didn't feel like making huge progress.
S9	Intermediate	It would be ideal to have uniformity with the English proficiency. That is the key to increase English proficiency.
S10	Basic	I consider it more beneficial to work with classmates who have similar English proficiency because the progress is consistent, learning is collective and more targeted feedback.
S11	Basic	I think it is better if they guide us. However, it is uncomfortable because we might make them fall behind or you feel a significant difference. It depends on who you are working with, if it is a person you feel comfortable with or not.
S12	Basic	I think it depends on the type of activity. When the job requires you to write, it is better to be with a partner who knows more about the rules. Also, learning is better.

As can be seen, more than half of the students preferred working with students whose language proficiency is the same or similar. The main reasons revolve around confidence and language output. S7 mentioned, "in my case, similar, because producing is more accessible for me and it allowed me to contribute more". The answer suggests that the student might feel more productive when working with classmates who possess similar language skills but less productive when working with those who are more proficient in the language. S2 indicated, "I believe that when the proficiency level is similar, it helps with confidence". The answer suggests that when mixing up levels, less proficient students might be at risk of shying away from actively participating when completing a task. Consequently, this strategy might affect task completion. S3 reinforced the previous

implication, “similar because learning and feedback occur in both directions”. The answer suggests that when working in a like-ability group, everyone involved is contributing more equally.

Another notable result is that only 3 out of 12 students leaned towards working with classmates whose language proficiency is different. S8 sees its value as higher proficiency level students would help with pronunciation or explanations of something. However, at the same time, S8 felt like not making a lot of progress when working with these students. Additionally, there were only 2 out of 12 who did not have a preference but considered both techniques to be beneficial. Lastly, the second column shows students’ self-perceived English proficiency. Thus, students seemed to prefer working with classmates who had a similar language proficiency regardless of their current English proficiency.

Differentiated Instruction

Throughout this course, differentiated instruction was employed to accommodate the diverse language proficiency levels, ensuring that each student could successfully complete tasks. Differentiated instruction was offered to students through differentiated worksheets tailored to individual students’ needs. Table 17 presents students’ opinions on differentiated worksheets tailored to their specific English language proficiency levels.

Table 17

Students’ Opinions on Differentiated Worksheets according to their Specific English

Language Proficiency

Students	Students’ Opinions
S1	Yes, it helps to improve according to our own level.
S2	Without a doubt, it is satisfying to see that despite the different English language proficiencies, we all benefit from the course.
S3	They were clear and had an appropriate level.
S4	I think it was beneficial. I can see the progress of the people who put effort into this. I think all their hard work paid off considering the general objective we had.
S5	I think they were very appropriate. It allows every person to progress at their own pace. Besides it helps lessen one the main challenges when learning another language, which is the fear of speaking the language.

- S6 Yes, because it facilitates the comprehension of the task we have been asked to complete.
- S7 Without a doubt, it was very beneficial. When it was implemented, it helped me complete the tasks.
- S8 The worksheets helped to guide the work we had to do.
- S9 I think it was a very useful learning material, it allows us to refer to the technicality such as vocabulary or phrases that would help with our fluency during oral tasks.
- S10 I progressed at my own pace which helped reduce stress.
- S11 Totally agree, it facilitates the learning process and the activities are very relevant.
- S12 In my case, I have a basic level. The worksheets were very resourceful because I could understand the task better and learn unknown vocabulary.
-

The data reveals that students responded positively, describing the worksheets as useful, beneficial, clear, appropriate, and resourceful. These responses suggested that differentiated worksheets had a favorable impact on students' learning by catering to their individual needs and abilities, thereby maximizing their learning potential in the classroom. Therefore, the implementation of differentiated worksheets appears to be an effective strategy for enhancing the learning experience for students across various English language levels.

Instructors' Perceptions on Differentiated Worksheets according to the Students' Specific English Language Proficiency. The following data points were collected through unstructured observations during class. When the strategy of using differentiated worksheets was initially implemented, the instructors observed that the main task posed an appropriate challenge for students at their respective proficiency levels. For instance, in Unit 3, Week 11's Main Task, students' speech samples demonstrated their ability to produce in the target language according to their proficiency levels (see Appendix E). While upper-level students were producing more fluently and independently, lower-level students required the assistance of sentence starters or incomplete scripts. In this task, higher-level students were able to complete it entirely, whereas lower-level students managed to complete two-thirds of it. Additionally, in Unit 3, Week 12's Main Task, lower-

level students were provided with more examples of useful language than higher-level students to explain how to solve, step by step, a trigonometry problem. A more detailed prompt seemed to have increased lower-level students' confidence and overall performance of the task. These results suggest that for lower-level students, the tasks were challenging yet achievable, potentially requiring additional time for full completion. The differentiated worksheets effectively catered to the diverse needs and abilities of students, fostering a conducive learning environment.

Furthermore, the findings in this section indicate that differentiated worksheets might foster cooperative learning and student engagement. During the breakout rooms in Unit 3, Week 11, lower-level students were no longer mere observers; instead, they actively participated and helped each other complete the task. Although they still relied on Spanish for some peer interaction, they were reading and showing an understanding of the guidelines in English and constructing mathematical explanations in the target language. Moreover, in Unit 2, Week 10's Main Task, which involved role-playing a teacher-student interaction, lower-level students were provided with a simplified script and vocabulary support, while higher-level students had to improvise and use more advanced language. Despite the challenges, lower-level students actively engaged in the task and supported each other, demonstrating their growing confidence and participation. The positive evolution of lower-proficiency students suggests that course design must consider including differentiated worksheets to be responsive to students' diverse needs in a mixed-ability class.

Another positive perception from the instructors based on their observations is that differentiated worksheets allowed them the opportunity to challenge upper-level students by increasing task demands in the class. For instance, in Unit 3, Week 11, higher-level students had to create a mathematical explanation independently without the aid of sentence starters, which were provided to lower-level students. This approach allowed

upper-level students to produce more creatively and independently. Additionally, it enabled instructors to provide targeted feedback on pronunciation, intonation, and other grammatical aspects of the language. Similarly, in Week 12, the main task included differentiated worksheets tailored to two different ability levels. Higher-level students were tasked with formulating explanations for determining the unknown side length of a triangle. Conversely, lower-level students were asked to complete and practice a pre-written script outlining the process of finding a missing side in a triangle. In this case, differentiated worksheets effectively catered to the diverse needs of students, fostering a challenging and enriching learning environment for those at higher proficiency levels. In addition, this differentiation allowed instructors to provide targeted feedback and support to students at their respective levels, ensuring a challenging and engaging learning experience for all.

One of the major drawbacks of differentiated worksheets, as noted by the instructors, is the time-consuming nature of planning and creating differentiated handouts for every lesson. Furthermore, during the lessons, instructors must exercise caution when providing instructions, as there are two or more different handouts or tasks in place, increasing the potential for student confusion. Effective implementation of differentiated worksheets requires meticulous preparation and clear communication from instructors to ensure a smooth learning experience for students across various proficiency levels.

Supervisor's Perceptions on Differentiated Worksheets according to Students' Specific English Language Proficiency. In a semi-structured interview with one of the practicum supervisors, she affirmed that differentiated worksheets are a useful technique that can facilitate student success. She highlighted that the definition of success is not uniform for all students, emphasizing the importance of catering to individual needs and abilities. The supervisor's perspective underscores the value of differentiated worksheets in creating an inclusive learning environment that promotes achievement for students across various proficiency levels.

The supervisor emphasized that the upper-level learners needed an additional challenge. She stated, “In the breakout rooms, I could see excellent negotiation of meaning; they were helping each other and even using some grammar structures you have not taught yet.” The supervisor acknowledged that mixed-ability groups are very common in ESP (English for Specific Purposes) courses and referred to the strategy of differentiated worksheets as highly appropriate for such courses. Drawing from her experience, she shared, “Whenever I have tried this technique in the past, it has been successful. With this technique, students were able to achieve their goals at their own level, and that is the point. Students’ success is the goal.” Furthermore, the supervisor addressed a potential concern that students might raise regarding this technique and mentioned, “Students might not be familiar with differentiated handouts in the math field. Therefore, it is important to discuss the usefulness with them. They need to know the value of this approach.” As shown, both the instructors and the supervisor agreed that differentiated worksheets benefit lower-level and upper-level students by targeting their individual needs to maximize learning. Another commonality found is that differentiated worksheets allow for creating a challenge for upper-level students who are ready to advance their learning. The supervisor’s insights further reinforce the effectiveness of differentiated worksheets in promoting student engagement, achievement, and a tailored learning experience in mixed-ability classrooms.

Summative Assessment

The summative assessment implemented in the course provided a comprehensive evaluation of students’ learning and achievement after the course. Table 18 compares students’ performance in the diagnostic test and the final speaking test.

Table 18

Comparison of Students’ Performance on Diagnostic Test and Final Speaking Test

Student	Performance
S1 Diagnostic Test	<p>S1 is partially successful pronouncing the mathematical vocabulary.</p> <p>S1 could read numbers and basic mathematical operations, but vocabulary range is lacking to explain more complex mathematical procedures.</p> <p>S1 lacked fluency and thorough explanations.</p>
S1 Final Speaking Test	<p>S1 was partially successful pronouncing the mathematical vocabulary.</p> <p>S1 was successful using the mathematical vocabulary and sequence markers when explaining mathematical procedures step by step.</p> <p>S1 explained the operations and solutions fluently and thoroughly.</p>
S2 Diagnostic Test	<p>S2 mispronounced most of the mathematical vocabulary.</p> <p>S2 was not able to complete some parts of the test.</p> <p>S2 has limited vocabulary to explain basic and advanced mathematical procedures and filled in this gap with Spanish words. E.g. “three dividido six plus five”.</p> <p>S2 lacked fluency</p>
S2 Final Speaking Test	<p>S2 was partially successful pronouncing the mathematical vocabulary.</p> <p>S2 was successful using the mathematical vocabulary and sequence markers when explaining mathematical procedures step by step.</p> <p>S2 explained the operations and solutions fluently and thoroughly.</p>
S3 Diagnostic Test	<p>S3 struggled pronouncing most of the mathematical vocabulary.</p> <p>S3 had limited vocabulary to explain basic and more advanced mathematical procedures.</p> <p>S3 lacked fluency.</p>

S3 Final Speaking Test	<p>S3 was partially successful pronouncing the mathematical vocabulary.</p> <p>S3 was successful using the mathematical vocabulary and sequence markers when explaining mathematical procedures step by step.</p> <p>S3 still lacked some fluency, some hesitation noted.</p>
S4 Diagnostic Test	<p>S4 mispronounced some of the mathematical vocabulary.</p> <p>S4 has limited vocabulary to explain more advanced mathematical procedures and filled in this gap with Spanish words.</p> <p>S4 was relatively smooth with some hesitation.</p>
S4 Final Speaking Test	<p>S4 could pronounce almost all mathematical vocabulary correctly, just a few minor errors noted.</p> <p>S4 was successful using a wide range of correct mathematics vocabulary and sequence markers when explaining a variety of mathematical procedures step by step.</p> <p>S4 explained the operations and solutions fluently and thoroughly.</p> <p>S4 seemed very confident when explaining mathematical procedures.</p>
S5 Diagnostic Test	<p>S5 could read numbers, several mathematical operators, symbols, and fractions. However, some pronunciation mistakes are noted. For example, S5 struggled pronouncing words that have the th sound or the lax vowel /ʌ/ as in the word subtract.</p> <p>S5 uses sequencing language to explain procedures step by step.</p> <p>S5 read exponential expressions incorrectly.</p> <p>S5 has limited vocabulary to explain more advanced mathematical procedures and filled in this gap with Spanish words.</p> <p>S5 uses incorrect vocabulary such as “divide” vs. “division”, “resolve” vs. “solve”, “left size” vs. “left side”</p>

	S5 lacked fluency.
S5 Final Speaking Test	<p>S5 could pronounce most mathematical vocabulary correctly, just a few minor errors noted.</p> <p>S5 used a wide range of correct mathematical vocabulary when explaining a variety of mathematical operations and solutions.</p> <p>S5 explained the operations and solutions fluently and thoroughly.</p>
S6 Diagnostic Test	<p>S6 could read numbers and few basic mathematical operators such as divide, multiply, subtract, and plus. However, pronunciation or intonation were not accurate.</p> <p>S6 could not read exponents, fractions, negative numbers.</p> <p>S6 code switched between English and Spanish to complete the explanation.</p> <p>S6 misused words such as less, minus, and subtract.</p> <p>S6 used incorrect vocabulary when explaining the procedures step by step. For example: S6 was trying to say “final result” and instead said “resulted finale”.</p> <p>S6 did not explain procedures throughout and fluently.</p>
S6 Final Speaking Test	<p>S6 pronounced most mathematics vocabulary correctly and had just some minor pronunciation mistakes in words such as power, solve, and operation.</p> <p>S6 was able to use correct mathematical vocabulary and sequencing language to explain mathematical procedures step by step.</p> <p>S6 could explain the operations and solutions thoroughly.</p> <p>S6 needs to become a little more fluent when explaining mathematical procedures.</p>

S7 Diagnostic Test	<p>S7 was unable to complete the task and skipped through most of the test because of lack of vocabulary.</p> <p>S7 just knew some numbers but mispronounced them. E.g. five. Three, add.</p>
S7 Final Speaking Test	<p>S7 was unsuccessful pronouncing most of the mathematical vocabulary.</p> <p>S7 was partially successful using mathematical vocabulary to explain the operations and solutions.</p> <p>S7 used sequencing markers successfully throughout the test.</p> <p>S7 sounded very hesitant.</p>
S8 Diagnostic Test	<p>S8 was unable to complete the task and skipped through most of the test because of lack of vocabulary.</p>
S8 Final Speaking Test	<p>S8 was partially successful pronouncing the mathematical vocabulary.</p> <p>S8 was successful using the mathematical vocabulary and sequence markers when explaining mathematical procedures step by step.</p> <p>S8 was very hesitant.</p>
S9 Diagnostic Test	<p>S9 could not pronounce most of the mathematical vocabulary correctly. E.g., three, half, result (syllable stress).</p> <p>S9 had limited vocabulary to explain the operations and solutions.</p> <p>Some errors noted: "8 on 10" instead of 8 over 10, "3 to 2" instead of three squared.</p> <p>S9 lacked fluency, unevenness caused by rephrasing, used fillers frequently.</p>
S9 Final Speaking Test	<p>S9 could pronounce almost all mathematical vocabulary correctly.</p>

- S9 was able to use correct mathematical vocabulary and sequence markers to explain the operations and solutions.
- S9 was partially fluent. In other words, the speech was relatively smooth with frequent fillers such as um/em.
- S10 Diagnostic Test S10 was partially successful pronouncing the mathematical vocabulary.
- S10 could read some mathematical vocabulary. However, some vocabulary is misused in phrases such as “divided from” or “the resulting is”
- S10 used sequence markers occasionally.
- S10 was not fluent, frequent unnatural pauses, and use of fillers such as um/em.
- S10 Final Speaking S10 was successful pronouncing the mathematical vocabulary.
- Test S10 was successful using a wide range of mathematical vocabulary and sequence markers when explaining mathematical procedures step by step.
- S10 explained the operations and solutions fluently and thoroughly.
- S11 Diagnostic Test S11 was unable to complete the task and skipped through most of the test because of lack of vocabulary.
- S11 Final Speaking S11 was unsuccessful pronouncing most of the mathematical
- Test vocabulary.
- S11 was partially successful using the mathematical vocabulary. The vocabulary range was still very limited.
- S11 frequently hesitated and had unnatural pauses.
- S11 used sequence markers successfully.

S12 Diagnostic Test	S12 was unable to complete the task and skipped through most of the test because of lack of vocabulary.
	S12 just knew some numbers but expressed that has a hard time pronouncing them.
S12 Final Speaking Test	S12 was partially successful pronouncing the mathematical vocabulary.
	S12 was successful using correct mathematics vocabulary and sequence markers when explaining the operations and solutions.
	S12 was partially fluent. In other words, the speech was relatively smooth with some hesitation.

As shown, by the end of the course, students demonstrated significant improvement in their communicative skills in the mathematics field. It is noteworthy that during the diagnostic test, regardless of their language proficiency, students relied on the use of Spanish to fill vocabulary gaps. However, in the final speaking test, none of them needed to resort to Spanish, indicating that both groups were able to expand their vocabulary repertoire in the mathematics field. In terms of pronunciation, there was still room for improvement, especially for lower-level students. However, the mistakes noted were minor and did not interfere with communication. These errors, such as vowel sounds that do not exist in their native language, are quite common among English language learners and are likely to improve over time as they progress in their language-learning process.

Another notable result is fluency. Compared to the diagnostic test, the intermediate students were able to explain mathematical procedures more fluently in the final test due to increased vocabulary acquisition and more accurate pronunciation. Although the

student with a basic level made significant gains in vocabulary and pronunciation, some hesitation and unnatural pausing were still present. Overall, the results suggest that the mixed-level strategies employed throughout the course may have contributed to achieving more successful outcomes in the final test compared to the diagnostic test, effectively addressing the diverse needs of students with varying proficiency levels.

Discussion

The purpose of this research paper was to assess the impact on task completion of the strategies proposed by recognized authors in the field and to draw practical implications for differentiating instruction in similar virtual ESP contexts with mixed proficiency level students. The results of this research provided supporting evidence on the effectiveness of the ESP course design, TBLT approach, scaffolding instruction, asynchronous and synchronous instruction, technology integration, cooperative learning, differentiated instruction, and formative and summative assessment. Teaching language learners with a mix of proficiency levels can present several challenges in any classroom environment. However, the results obtained after administering the designed instruments and observations revealed that adequate planning of the strategies according to the students' abilities and needs yielded satisfactory results for the teaching-learning process in this context.

Formative Assessment

Formative assessments such as the practice tasks reflected in the weekly main tasks seemed to have an impact on task completion in the summative assessments. Research indicates that formative assessments like observations, discussions, and practice tasks, conducted throughout lessons, provide ongoing feedback to students and enable teachers to monitor learning and adjust instruction and scaffolding in response to individual progress and with mixed proficiency levels in mind (Echeverria, 2017). The unstructured observations during the course helped the instructors to determine the

appropriate time to level up students' tasks. As seen in Table 9, the syllabus included scaffolding of tasks from structured tasks such as a scripted role-play to less structured tasks like students constructing a mathematical explanation on their own. The ongoing unstructured observations provided insights at key times during the course that not all students were ready to transition to more complex tasks and, therefore, pointed to the need for implementing other strategies such as differentiated worksheets and homogenous grouping, which are further explained in the next sections.

TBLT and Balancing Synchronous and Asynchronous Instruction

According to the assessment survey for self-reflection and course improvement results, students showed a positive perception of the course design. Research studies have found that the balance between synchronous and asynchronous instruction can potentially enhance diverse students' linguistic development, content knowledge acquisition, and overall satisfaction in mixed-level ESP courses (Cunningham, 2014; Satar & Özdener, 2008). In Table 11, S3 and S5 mentioned that they would have liked to have more time to dedicate to the course. The students were likely referring to the asynchronous time, considering they had two and zero absences throughout the course respectively. They probably saw the value of the tasks assigned during asynchronous time and wished they could have dedicated more time to them, as it would have been beneficial for their linguistic development.

The asynchronous session was mostly focused on targeted grammatical and phonetic aspects of the language. For example, in Week 2, Lesson 1, as part of the asynchronous session, students had to watch videos about the pronunciation of the sounds "shun" [ʃ] & "zhun" [ʒ]. After watching the video, students had to practice a list of words and get ready to pronounce them correctly in the next class. In Week 3, Lesson 2, the instructor started the class by reviewing the list of words and providing feedback. Some students seemed to have practiced the words, while others did not. Students who

completed the asynchronous session were most likely more successful in completing tasks in the next lesson compared to those who did not and still struggled with the pronunciation of the words, as observed in the class recordings. The data suggest that course design must continue to consider asynchronous instruction to help students reinforce content independently and be more prepared to achieve task completion in the following class. Finally, one way to ensure task completion during the asynchronous session is to consider the task as a graded portion in the course evaluation.

Technology Integration

As mentioned in the results section, student-teachers employed a variety of technology tools and platforms throughout the course, and this seemed to have a positive impact on students' perceptions towards the course. It also seems to have fostered students' engagement and potentially task completion. According to research, technology integration can provide scaffolding to support task completion when teaching mixed-proficiency ESP learners (Ernst-Slavit & Wenger, 2006; González-Lloret, 2020; Song, 2016; Subrahmanyam Vellanki & Bandu, 2021). The following is an example of technology integration in the course and how it seemed to have contributed to students' engagement, scaffolding, and task completion. In Week 3, the instructor displayed a PowerPoint Presentation, where they read the description of some geometric shapes, and students volunteered to choose the shape that belonged to the description provided. The presentation included pictures of the geometric shapes, their names, and definitions, serving as a useful visual resource for students to learn new terminology.

According to unstructured observations, the students seemed to be engaged in the task and had the opportunity to practice the pronunciation of new terminology that they would use throughout the lesson. The student-teacher also provided immediate feedback when students mispronounced the vocabulary. Consequently, the task provided scaffolding to support the completion of the subsequent, more demanding tasks, where

students were going to be required to use the new terminology in context. In this regard, Yilmaz (2011) highlights that multimedia scaffolds like videos and slide presentations are also helpful in the online environment. Technology is appealing to the new generation of students, and thus, teachers can see technology as a vehicle that has an enormous potential to bring scaffolding and learning to students in a variety of ways while keeping them engaged.

TBLT Scaffolding Instruction Principle

In Table 18, the data pointed to students achieving task completion in the final test as well as their linguistic improvement, more specifically in the area of vocabulary acquisition. It is worth highlighting that most learners were initially categorized as beginners except for three students, who were considered intermediate language learners. Regarding these conditions, the literature recommends that teachers employ varied teaching methods and scaffolding techniques to support beginning-level students in an online context (Scrivener, 2012; Thamarana, 2016). According to the information recovered from unstructured observations, scaffolded instruction was consistently implemented in every lesson, and this might have contributed to students' success in the final speaking test. One example is from Unit 2, Week 5: as part of the warm-up, the student-teacher started the lesson by reviewing previously learned vocabulary to activate students' background knowledge. During the pre-task stage, the student-teacher provided new vocabulary input through videos and a handout including a visual guide on how to read math. Students had the chance to practice the new vocabulary in small groups by reading an equation, referring to the guide in case they forgot how to read a mathematical symbol. As the main task, students were engaged in a semi-scripted role-play, where they had to first complete the script with the new vocabulary and then practice the role-play with a classmate.

The students-teachers were observed providing immediate feedback along the way. Scrivener (2012), suggests that learners may need extra support to successfully complete meaningful tasks, and he suggests that teachers can provide this support through scaffolding techniques such as modeling, providing language frames (pre-constructed phrases or sentence structures), giving feedback, asking leading questions, and reviewing key vocabulary before the task. In this lesson, there were several instances where scaffolding was present. There was modeling through the videos displayed of mathematics professors explaining mathematics content. There were language frames through semi-scripted role-play and the guide on how to read math (See Appendix F). There was also reviewing of key vocabulary in the warm-up and pre-task stages, as well as feedback provided by student-teachers. As learners progressed throughout the course, some of them did not need additional support and were able to complete tasks more independently. For example, in Unit 3, Weeks 11 and 12, the main task included differentiated worksheets so that higher-level students were able to engage in more challenging tasks like constructing the explanation of finding a missing side in a triangle on their own, while lower-level students had to complete and practice a script about finding a missing side in a triangle. This differentiation ensured that each group could engage meaningfully with the content at their own level of language proficiency. In conclusion, TBLT scaffolding instruction appears to have assisted learners with task completion in class and in the summative assessments.

Cooperative Learning

As seen in the results section, the students', supervisor's, and instructors' perspectives-all seem to agree that dividing the group by proficiency level to facilitate differentiated work was the most preferable and beneficial strategy implemented in the course. According to Scrivener (2012), the strategy might come with important drawbacks such as loss of class identity or upper-level students showing a sense of superiority

against lower-level students. However, this was not the case in this course thanks to the valuable recommendations received by the supervisor throughout the course, which highlights the importance of communicating with the students the value and purpose behind the implementation of the strategies mentioned above. Therefore, if handled this way, clear communication might prevent students from constructing misconceptions about these strategies. In addition, the ability to build a safe environment in the classroom, one full of trust and respect as found in many of the students' answers in the surveys, also played a huge role in the implementation of such strategies.

Based on previous results, students with a basic level agreed on three important aspects when working in homogenous grouping: targeted feedback, collective learning, and consistent progress. The answer suggests that when students are grouped with peers who possess a similar level, it may boost their confidence to take more risks to contribute or participate in the task. There is nothing to lose because they are all in a vulnerable position. They rely on helping each other to complete the task, and that most likely gives them a sense of accomplishment at the end. Thus, they might perceive progress as more consistent. The opposite might happen when working with classmates whose level of proficiency is higher. The affective filter might increase in that scenario. Besides, the amount of production is most likely imbalanced. Students with a higher level are probably producing more compared to those with a lower proficiency, who might feel hesitant to contribute. Therefore, they rely on learning from them, which is good. However, it might leave lower students with a sense of failure because they could not produce as much as the others. Nevertheless, some of the students also find heterogenous grouping beneficial for their own learning process. As shown in the results section, upper-level students seemed to benefit from helping others, which reinforced their own learning. Similarly, less proficient students reported learning from more proficient students. According to Johnson and Johnson (1999), keeping the class as a single mixed-level group also has its benefits

by supporting and learning from one another through peer modeling, scaffolding, and feedback. Therefore, varying the grouping techniques throughout the lesson is fundamental to better satisfy students' needs. In other words, although most students in this group prefer working with classmates whose proficiency level is similar, research recommends a well-thought-out balance between homogenous and heterogeneous grouping for a mixed-level class to thrive.

Differentiated Instruction

Taken together, our findings indicate that the variety of strategies and techniques implemented throughout this course helped all students with task completion at their own level. The results from the final speaking test show that students were able to acquire more mathematics vocabulary as well as enhance its pronunciation regardless of their English proficiency level. Using appropriate mathematics vocabulary to explain mathematical procedures was one of the main tasks in the course. Upper-level students were able to construct their own explanations regarding mathematical procedures more independently and fluently compared to lower-level students, who were able to produce within a more structured type of task. Still, regardless of the different proficiency levels in the group, both basic and intermediate students achieved the course goal, which was task completion at their own level. This outcome points to the conclusion that differentiated instruction, such as providing simplified content and tasks for less proficient learners while integrating complex and challenging materials to push advanced students, can have positive outcomes for a mixed-level classroom.

Summative Assessment

Brown and Abeywickrama (2018) note that summative assessment, like projects, presentations, and tests, evaluates overall learning outcomes and proficiency gains. The final speaking test was a significant instrument that provided the course design team with valuable insights in terms of students' learning growth. As can be seen in Table 18, each

learner showed individual progress. Some learners moved from partially pronouncing mathematical vocabulary successfully in the diagnostic test to consistently pronouncing mathematical vocabulary successfully in the final speaking test. Others stayed in the partially successful category when pronouncing mathematical vocabulary but showed gains in vocabulary usage that allowed them to complete the task, compared to the diagnostic test where they were not able to complete it. Therefore, it is noticeable that not all students fully achieved the targeted proficiency by the end of the course; however, all of them demonstrated their unique progression towards the course's common goal: task completion at their own level.

Conclusions

The data collected from the surveys, the semi-structured interview with the supervisor, the unstructured observations, and the diagnostic and final tests provided valuable insights to address the research questions. The main research question sought to understand how differentiated instructional strategies impact task completion in a mixed-proficiency virtual ESP class for mathematics professors. The results strongly suggest that the implementation of differentiated strategies, such as grouping students by proficiency level and using tailored instructional materials, positively impacted students' task completion and overall performance.

The first supporting research question asked which strategies and techniques most effectively supported students of varying proficiency levels in completing ESP tasks in a virtual environment. The results show that students preferred working in groups with peers whose proficiency levels were similar, as this fostered a more comfortable and confident environment for participation. This strategy seemed to enhance students' motivation and willingness to take risks, contributing to more active engagement in the activities. Additionally, the use of differentiated worksheets, targeting specific student needs and

abilities, was reported to have a significant positive impact on learning, helping students progress at their own pace while maximizing their potential.

The focus of the second research question was how students perceived the effectiveness of differentiated instruction strategies in supporting their task completion. Students expressed satisfaction with the strategies implemented throughout the course. Their feedback indicated that the grouping by proficiency and the tailored worksheets helped them better understand the course material and improve their performance. This suggests that students not only perceived these strategies as effective but also appreciated their role in supporting task completion and fostering a more inclusive learning environment.

Lastly, the third research question looked into how supervisors evaluated the implementation and effectiveness of differentiated instruction strategies in supporting mixed-proficiency students' task completion. Both the supervisors and course designers agreed on the effectiveness of the strategies. This consensus was reflected in the significant improvement observed in students' performance between the initial diagnostic and the final speaking test. The students showed considerable progress in using mathematical vocabulary, and their ability to engage in listening, reading, and speaking activities improved, with speaking being the most noticeable area of growth.

In conclusion, the results of this study align with the research question and supporting objectives by providing evidence that differentiated instruction strategies indeed supported mixed-proficiency students in a virtual ESP context. These strategies, particularly grouping by proficiency level and using differentiated materials, revealed themselves to be effective in enhancing task completion and engagement in the ESP course for mathematics instructors, thus improving overall learning outcomes. The findings of this study provide valuable insights for course designers seeking to implement

differentiation in similar educational settings, suggesting that such strategies can effectively support students' progress, regardless of their initial proficiency level.

Limitations

The most salient limitation of this study concerns the timeline for the development of the research project as well as time constraints. It would have been ideal and perhaps more beneficial to have had the research questions and the literature review drafted before the start of the ESP course. First of all, the process would have been less overwhelming for instructors, as the practicum itself requires a lot of reading, preparation, and planning before every class, especially if instructors are novice in such a broad and complex field as mathematics. Therefore, carrying out a research project simultaneously was quite a challenge. As mentioned before, having at least the research questions and literature review ahead of time would have brought more clarity to the instructors in terms of course goals and planning from the very beginning. In other words, it would have been ideal to integrate it during the course design process in the first semester. For instance, just as the units and content were planned and distributed in advance, the strategies and techniques the instructors intended to use with this group would have been mapped out or tentatively scheduled from the beginning to the end of the course. Consequently, the delayed implementation of some of the strategies hindered the student-teachers from collecting more insightful data for this paper since the start of the course.

Time constraints were another limitation. The student-teachers' lack of expertise in teaching an ESP course, especially related to the field of mathematics, also contributed to time constraints at the beginning of the course. For example, some pre-tasks took longer than expected for students to complete; the student-teachers were also including two pre-tasks at the beginning of the course making it difficult for them to complete the task cycle. After thoughtful consideration, the team decided to plan just for one pre-task in order to be able to complete the task cycle.

Task completion was a significant part of this research: strategies were being implemented at the moment to support task completion and sometimes the student-teachers could not provide them with additional time to complete them because they could have interfered with the completion of the task cycle which was part of the student-teacher evaluation. Therefore, some results show that students were not able to complete the task, but the student-teachers believe it was mostly because of time constraints and not because the strategy to support task completion was not effective. Student-teachers believe that students would have been able to complete the tasks if they had had additional time. Therefore, time constraints became a factor when drawing implications in the data analysis section.

Recommendations

Based on the comprehensive evaluation of the strategies and techniques implemented in the virtual English for Specific Purposes (ESP) course for mathematics professors, several recommendations can be made to enhance the effectiveness of differentiated instruction in similar mixed-proficiency contexts.

First, it is crucial to conduct a thorough needs analysis at the outset of the course to thoroughly understand students' current proficiency levels, learning preferences, and specific language needs within the target discipline. This data should then directly inform the course design, including the selection and sequencing of content, tasks, and assessments. By aligning instruction to the unique requirements of the learner population, instructors can create a more personalized and impactful learning experience for students across proficiency levels, just as student-teachers effectively did for this course.

Next, the judicious integration of both synchronous and asynchronous instructional modes is recommended. The findings suggest that the balanced approach, where synchronous lessons provide opportunities for real-time interaction and feedback, while self-paced asynchronous tasks allow students to work at their own pace, can effectively

address the diverse needs of mixed-level learners. Instructors should strategically plan the distribution of content, practice, and assessment between the two modalities to optimize learning.

Furthermore, the consistent implementation of scaffolding techniques, such as modeling, providing language frames, giving feedback, and activating background knowledge, emerged as a key factor in supporting task completion, particularly for lower-proficiency students. Instructors should carefully monitor student progress and adjust the level of support accordingly as learners develop greater independence. Additionally, the carefully-planned use of differentiated worksheets, where content and tasks are tailored to proficiency levels, can foster an inclusive learning environment that challenges advanced students while offering appropriate support to those at lower levels.

Finally, a balanced approach to formative and summative assessment, incorporating both ongoing feedback and final evaluations of learning outcomes, is recommended. Formative assessment enables instructors to gauge individual progress and make timely instructional adjustments, while summative assessments with differentiated criteria allow for a fair and accurate evaluation of students' learning gains. Additionally, incorporating student self-assessment strategies can promote learner autonomy and awareness of their own progress.

By implementing these recommendations, instructors can create a more equitable and effective virtual ESP learning environment that caters to the diverse needs of mixed-proficiency students, ultimately enhancing their language development and task completion within the target discipline.

Three key areas warrant further research to address limitations and unexplored questions from this study of differentiated instruction in virtual ESP teaching. First, longitudinal studies should examine the sustained impact of differentiated strategies on student progress over extended periods, providing deeper insights into long-term

effectiveness across proficiency levels. Second, research is needed to determine optimal ratios and timing for homogeneous versus heterogeneous grouping in virtual ESP contexts, as this study revealed student preferences for similar-level groupings but left questions about ideal implementation. Finally, future studies should investigate how to effectively scale these differentiated instruction strategies for larger virtual ESP classes, as the current findings are based on a relatively small group. Such research could explore techniques for managing multiple proficiency levels efficiently while maintaining personalized support in larger virtual learning environments. These investigations would help address current knowledge gaps and strengthen our understanding of effective differentiation strategies in virtual ESP instruction.

References

- Basturkmen, H. (2015). *Developing courses in English for specific purposes*. Springer.
- Bielousova, R. (2017). Developing materials for English for specific purposes online course within the blended learning concept. *Tem Journal*, 6(3), 637-642.
- Brown, H. D., & Abeywickrama, P. (2018). *Language assessment: Principles and classroom practices* (3rd ed.). Pearson Education.
- Bui, H. (2022). Students' and teachers' perceptions of effective ESP teaching. *Heliyon* 8(9), 1-2. <https://doi.org/10.1016/j.heliyon.2022.e10628>
- Burt, M., Peyton, J. K., & Schaetzel, K. (2008). Working with adult English language learners with limited literacy: Research, practice, and professional development. *Center for Applied Linguistics*.
- Crandall, J., & Kaufman, D. (2002). *Content-based instruction in higher education settings*. Teachers of English to Speakers of Other Languages, Inc.
- Cunningham, U. (2014). Teaching English pronunciation online to Swedish primary-school teachers. In E. Waniek-Klimczak & M. Pawlak (Eds.), *Teaching and researching the pronunciation of English: Studies in honour of Włodzimierz Sobkowiak* (pp. 63-76). Springer International Publishing.
- DeCapua, A., & Marshall, H. W. (2015). Implementing a mutually adaptive learning paradigm in a community-based adult ESL literacy class. In M. G. Santos & A. Whiteside (Eds.), *Low educated second language and literacy acquisition. Proceedings of the Ninth Symposium*, 9(1), 151–171. <https://doi.org/10.5281/zenodo.8022534>
- Dudley-Evans, T., & St John, M. J. (1998). *Developments in English for specific purposes: A multi-disciplinary approach*. Cambridge University Press.
- Dung, D. T. H. (2020). The advantages and disadvantages of virtual learning. *IOSR Journal of Research & Method in Education*, 10(3), 45-48.

- Echevarria, J., Vogt, M. E., & Short, D. (2017). *Making content comprehensible for English learners: The SIOP Model* (5th ed.). Pearson
- Ernst-Slavit, G., & Wenger, K. J. (2006). Teaching in the margins: The multifaceted work and struggles of bilingual paraeducators. *Anthropology & Education Quarterly*, 62-82.
- Galletta, A. M. (2013). *Mastering the semi-structured interview and beyond*. NYU Press.
<https://doi.org/10.18574/nyu/9780814732939.001.0001>
- González-Lloret, M. (2020). Collaborative tasks for online language teaching. *Foreign Language Annals*, (53), 260-269.
- Herrera, S. G., & Murry, K. G. (2006). Mastering ESL and bilingual methods: Differentiated instruction for culturally and linguistically diverse (CLD) students. *Education Review*.
- Huang, Y. C., & Liao, L. C. (2015). A study of text-to-speech (TTS) in children's English learning. *Teaching English with Technology*, 15(1), 14-30.
- Hyland, K. (2006) *English for Academic Purposes*. Routledge.
- Hyland, K., & Hamp-Lyons, L. (2002). EAP: Issues and directions. *Journal of English for Academic Purposes*, 1(1), 1-12.
- Johnson, D. W., & Johnson, R. T. (1999). Making cooperative learning work. *Theory into practice*, 38(2), 67-73.
- Lee, J., & Van Patten, B. (2003). *Making communicative language teaching happen* (2nd ed.). McGraw-Hill Education.
- Lightbown, P. M. (2016). SLA research and foreign-language teaching. *Handbook of Research in Second Language Teaching and Learning*, 103-116.
- Long, M. H. (Ed.). (2005). *Second language needs analysis*. Cambridge University Press.
- McKechnie, L. E. F. (2008). Observational research. *The Sage Encyclopedia of Qualitative Research Methods*, 573–577.

- Lopez, V., & Whitehead, D. (2012). Sampling data and data collection in qualitative research. In Z. Schneider & D. Whitehead (Eds.), *Nursing & midwifery research: Methods and appraisal for evidence-based practice* (4th ed., pp. 123 – 140). Elsevier-Mosby.
- Ortiz, A. A., & Fránquiz, M. E. (2017). Co-editors' introduction: The central role of advocacy in ensuring excellence in education for English learners. *Bilingual Research Journal*, 40(3), 241-245.
- Öztürk, M. (2021). Asynchronous online learning experiences of students in pandemic process: Facilities, challenges, suggestions. *Turkish Online Journal of Qualitative Inquiry*, 12(2), 173-200.
- Richards, J. C., & Rodgers, T. S. (2014). *Approaches and methods in language teaching*. Cambridge University Press.
- Roy, A., Guay, F., & Valois, P. (2013). Teaching to address diverse learning needs: Development and validation of a differentiated instruction scale. *International Journal of Inclusive Education*, 10(3), 1186-1204.
<https://doi.org/10.1080/13603116.2012.743604>
- Satar, H. M., & Özdener, N. (2008). The effects of synchronous CMC on speaking proficiency and anxiety: Text versus voice chat. *The Modern Language Journal*, 92(4), 595-613.
- Scrivener, J. (2012). *Classroom management techniques*. Cambridge University Press.
- Slavin, R. E. (1980). Cooperative learning. *Review of educational research*, 50(2), 315-342.
- Slavin, R. E. (1996). Research on cooperative learning and achievement: What we know, what we need to know. *Contemporary educational psychology*, 21(1), 43-69.
- Standards for Educational and Psychological Testing. (2014). *American Educational Research Association*.

- Starman, A. B. (2013). The case study as a type of qualitative research. *Journal of Contemporary Educational Studies*, 30.
- Subrahmanyam Vellanki, S., & Bandu, S. (2021). Engaging students online with technology-mediated task-based language teaching. *Arab World English Journal (AWEJ) Special Issue on Covid 19*.
- Thamarana, S. (2016). Role of e-learning and virtual learning environment in English language learning. *Teaching English Language and Literature: Innovative Methods and Practices, ELTAI Tirupati*, 61-62.
- Tomlinson, B. (2012). Materials development for language learning and teaching. *Language teaching*, 45(2), 143-179.
- Vogt, K., & Kantelinen, R. (2013). Vocationally oriented language learning revisited. *ELT journal*, 67(1), 62-69.
- Yilmaz, C. (2011). Teachers' perceptions of self-efficacy, English proficiency, and instructional strategies. *Social Behavior and Personality: An international journal*, 39(1), 91-100.

Appendix A

University of Costa Rica
Master's Program in TEFL
PF-0309 Course Design
English for Mathematics Teaching

Entrevista con representantes la sección de Enseñanza de la Matemática

El propósito de este documento es recopilar información sobre el estudiantado de la sección de Enseñanza de la Matemática de la Universidad de Costa Rica, Sede de Occidente, para el diseño e implementación del curso de inglés con fines específicos como parte de la Maestría Profesional en la Enseñanza del Inglés.

1. ¿Cuántos estudiantes podríamos tener en el curso?
2. ¿Cuál sería el promedio de edad de los estudiantes?
3. ¿Tiene conocimiento de dónde es la mayoría de estudiantes del programa?
4. ¿Cuál es el nivel académico de los estudiantes (año de carrera)?
5. ¿Cuál es su impresión del nivel de inglés de los estudiantes (básico, intermedio, avanzado)?
6. ¿La mayoría de estudiantes son graduados de colegios públicos o privados?
7. ¿El enfoque del curso debe ser académico o profesional?
8. ¿Qué expectativas tienen con respecto al curso de inglés que vamos a diseñar e implementar?
9. En cuanto a las necesidades del uso del inglés tanto en cursos universitarios como en el campo laboral, ¿tiene alguna idea de cuáles podrían ser las principales deficiencias que tanto los estudiantes como los profesores del programa podrían tener con respecto al uso del inglés?
10. De los cursos del plan de estudios, ¿cuáles requieren contacto con el inglés?
11. ¿Los estudiantes han llevado otros cursos de idioma como parte de su programa de estudios?
12. ¿Cuáles tareas o actividades específicas podrían desempeñar en su futuro profesional?
13. ¿Cuáles destrezas debemos priorizar en el curso (comprensión auditiva, expresión oral, comprensión lectora, expresión escrita)?
14. ¿Con qué frecuencia los estudiantes deben leer y entender artículos en inglés?
15. ¿Con qué frecuencia los estudiantes deben escribir ensayos en inglés?
16. ¿Con qué frecuencia los estudiantes deben hacer investigaciones en inglés?

17. ¿Con qué frecuencia los estudiantes deben participar en charlas y ponencias en inglés?

Appendix B

Cuestionario

Este cuestionario ha sido diseñado con el propósito de recolectar información para el diseño e implementación de un curso de los estudiantes de la sección de Enseñanza de la Matemática de la Universidad de Costa Rica, Sede de Occidente. Esto forma parte de un proyecto de la Maestría Profesional de la Enseñanza del Inglés como Lengua Extranjera. La información recolectada será de uso confidencial y se empleará únicamente para este proyecto de investigación.

I. Información personal

Complete la siguiente información personal.

Nombre: _____

Correo electrónico: _____

Número de teléfono: _____ Año de carrera: _____

Marque la categoría que corresponda a su edad actual.

_____ 18-20 _____ 21-29 _____ 30-39 _____ mayor de 40

II. Experiencia en el idioma inglés

1. ¿Cuáles de las siguientes opciones reflejan su experiencia al estudiar inglés anteriormente?

_____ educación primaria

_____ educación secundaria

_____ clases privadas

_____ instituto

_____ aprendizaje autónomo

_____ otro (especifique) _____

2. ¿Cómo considera usted su nivel de inglés para comunicarse de manera oral?

_____ Excelente _____ Muy bueno _____ Aceptable _____ Deficiente

3. ¿Cómo considera usted su nivel de inglés para comunicarse de manera escrita?

_____ Excelente _____ Muy bueno _____ Aceptable _____ Deficiente

III. Actitud ante un curso de inglés

De las siguientes afirmaciones marque con una (x) si considera que es verdadero, falso, o no aplica.

	Verdadero	Falso	No aplica
1. Disfruto ser parte de un curso de inglés.			
2. Considero que aprendo inglés con facilidad.			
3. Aprender inglés para mí es sólo una obligación.			
4. Participar activamente en clases es de mi agrado.			

5. Me siento atraído por estar en contacto con el idioma dentro y fuera de clases (películas, música, lectura).			
6. Considero que el inglés es necesario para mis estudios actuales.			

Destrezas del idioma

Marque con una (X) una de las opciones brindadas por cada destreza.

Escritura

<input type="checkbox"/> Puedo escribir un párrafo sencillo en inglés con facilidad.
<input type="checkbox"/> Puedo escribir un ensayo o informe avanzado en inglés con facilidad.
<input type="checkbox"/> Ninguna de las anteriores.

Lectura

<input type="checkbox"/> Puedo leer en inglés con facilidad sin hacer uso de un diccionario.
<input type="checkbox"/> Leer en inglés me es difícil por lo que necesito hacer uso de un diccionario.
<input type="checkbox"/> Ninguna de las anteriores.

Habla

<input type="checkbox"/> Logro expresar mis ideas con facilidad cuando hablo en inglés.
<input type="checkbox"/> Se me dificulta expresar mis ideas cuando intento hablar en inglés.
<input type="checkbox"/> Ninguna de las anteriores.

Escucha

<input type="checkbox"/> Puedo entender inglés con facilidad cuando escucho a otra persona hablarlo.
<input type="checkbox"/> Se me dificulta entender inglés cuando escucho a otra persona hablarlo.
<input type="checkbox"/> Ninguna de las anteriores.

Lea las siguientes destrezas e indique en una escala del 1 al 4 el nivel de dificultad, donde 4 quiere decir que es muy difícil y 1 que es muy fácil para usted.

Destreza	1	2	3	4
a. Entender audios o videos relacionados con las matemáticas				
b. Entender charlas o ponencias relacionadas con las matemáticas				

c. Participar en conversaciones relacionadas con las matemáticas				
d. Pronunciar correctamente vocabulario relacionado con las matemáticas				
e. Entender problemas matemáticos en inglés				
f. Explicar problemas matemáticos en inglés				
g. Otra (especifique): _____				

IV. Preferencias y estilos de aprendizaje

Lea las siguientes opciones. Utilice del número 1 al 10 para indicar cuáles son de su mayor y menor preferencia.

- Participar en discusiones de clase _____
- Participar en juegos y dinámicas de clase _____
- Exponer temas relevantes en clases _____
- Aprender a través de apoyos visuales (diapositivas, imágenes, videos) ____
- Aprender a través de audios o grabaciones _____
- Aprender a través de lecturas y análisis de textos _____
- Juegos de roles (profesor de matemáticas & estudiante) _____
- Trabajar en grupos _____
- Trabajar individualmente _____
- Otra (especifique): _____

V. Uso del inglés en el desarrollo profesional

Lea las siguientes destrezas en inglés e indique cuáles tienen mayor prioridad para usted enumerándolas del 1 al 7.

- Aprender vocabulario relacionado con las matemáticas _____
- Pronunciar correctamente vocabulario relacionado con las matemáticas _____
- Participar de forma fluida en conversaciones relacionadas con las matemáticas _____
- Escribir textos y ensayos sobre temas relacionados con las matemáticas _____
- Entender con facilidad operaciones matemáticas explicadas por hablantes nativos _____

f. Explicar de forma fluida operaciones matemáticas _____

g. Otra (especifique) _____

VI. Expectativas del curso

Indique enumerando del 1 al 4 ¿Qué le gustaría aprender en el curso de ESP (inglés con fines específicos) para ayudarlo a mejorar sus destrezas en el idioma?

_____ Habilidades de escritura

_____ Habilidades de

comunicación oral

_____ Comprensión auditiva

_____ Comprensión lectora

¡Gracias por su tiempo!

Appendix C

University of Costa Rica
Master's Program in TEFL
PF-0309 Course Design
English for Mathematics Teaching

Diagnostic Test

General Instructions

The diagnostic test intends to gather information on your estimated level of English proficiency in listening, speaking, and reading. You will have 60 minutes to complete the test with the required information.

Read the specific instructions carefully for each section of the test. Ask the test administrator if you have questions.

Estimated time:

Part I. Listening: 20 min

Part II. Reading: 20 min

Part III. Speaking: 15 min

I Part. Listening. You will listen to a math teacher explaining two different procedures about fractions. You will listen to each procedure twice.

A. Read the statements below. Then listen attentively to the first explanation about adding and subtracting fractions. Complete the following statements with the missing words.

1. The word used in the audio to describe the numerator of a fraction is _____ number.
2. The word used in the audio to describe the denominator of a fraction is _____ number.
3. The word used in the audio to name fractions with the same denominators is _____ fractions.

4. The word used in the audio to name the fractions with different denominators is _____ fractions.
5. The expression used in the audio to describe the process of changing fractions to add them is called _____.

B. Read the statements below. Then listen carefully to the second explanation about the procedure for finding Least Common Denominators (LCD). Finally, put the following steps in order from 1 to 6 according to the audio.

_____ Continue making multiples until you find the same multiple (or a common multiple) for both denominators.

_____ Multiply the fractions to get new equivalent fractions with the same denominator.

_____ Start making multiples of each denominator

 2 Create a small table to organize the multiples of the two denominators.

_____ Add the numerators and keep the common denominator to add the fractions.

_____ Find the whole fractions that should be multiplied by the original fractions.

II Part. Reading. Read the lesson below about Writing and Interpreting Expressions.

Then, answer the questions related to the lesson.

LESSON 1

Write and Interpret Expressions



Coached Instruction

- Understand Colin's method for solving the problem below.

Which expression represents the problem below?

6 less than 10 times 2

- A. $6 \times 10 - 2$
 B. $(10 \times 2) - 6$
 C. $6 - (10 \times 2)$
 D. $10 \times 6 - 2$

COLIN'S METHOD

I know that "10 times" 2 means multiplication so I can write 10×2 .

I can group 10×2 as (10×2) .

I know that "6 less than" means subtraction so I can write $- 6$.

I can combine the parts of the expression to write $(10 \times 2) - 6$.

Choice B is correct.

Taken from Coachdigital.com

- A. Read the following questions and circle the right answer. (5pts., 1pt. each)
1. Which expression means 6 less than 15?
 - a. $15 - 6$
 - b. $15 \div 6$
 - c. 15×6
 - d. $15 + 6$
 2. Which expression represents "subtract 6 from 18, then divide by 3"?
 - a. $18 \div (6 - 3)$
 - b. $18 - (6 \div 3)$
 - c. $18 - 6 \div 3$
 - d. $(18 - 6) \div 3$

3. Which expression means “add 12 and 9, then multiply by 6”?
- $6 \times 12 + 9$
 - $6 \times (12 + 9)$
 - $6 \times 9 + 12$
 - $6 + (12 \times 9)$
4. Which expression represents “add 465 and 371, then multiply by 2”?
- $2 \times 465 + 371$
 - $465 + 2 \times 371$
 - $2 \times (465 + 371)$
 - $2 \times 465 - 371$
5. Rachel bought and downloaded 6 songs on Friday and 8 more songs on Saturday. Each song cost \$2. Which expression represents the amount of money that Rachel paid?
- $6 \times 8 \times 2$
 - $6 \times 8 + 2$
 - $2 \times 6 + 8$
 - $2 \times (6 + 8)$

- B. Read the following classroom activity about the volume of a cylinder. Organize the procedures in logical order by filling in the blanks from 1-7. Two of them are already solved for you as an example. (5pts., 1pt. each)

	In the warm-up, students recall that a circle’s area can be determined given its radius or diameter.
	The next lesson will provide opportunities to practice these skills and solve related problems.

1	In this lesson students learn that the volume of a cylinder is the area of the base times the height, just like a prism.
	Finally, students compute the volume of a cylinder by multiplying the area of its base by its height.
	This is accomplished by considering 1-unit-tall layers of a rectangular prism side by side with 1-unit-tall layers of a cylinder. After thinking about how to compute the volume of specific cylinders, students learn the general formulas $V=Bh$ and $V=\pi R^2h$.
6	A volume expressed using the exact number π versus the same volume computed using 3.14 as an approximation for π is discussed.
	Students also become familiar with what is meant by <i>radius</i> and <i>height</i> as those terms apply to cylinders.

Adapted from illustrativemathematics.org

III Part. Speaking.

Warm up: Why did you decide to study math? Why did you decide to teach math?

A. Read aloud the following mathematical operations and their respective solutions.

1.	$12 \div 6 + 5 = 7$
2.	$40 \div 4 \times 5 - 15 = 35$
3.	$(5 - 3) + (6 \times 2) = 14$
4.	$(3^2 \times 4) - 6 = 30$
5.	$\frac{8}{10} + \frac{7}{10} = \frac{3}{2}$

6.	$\frac{1}{2} \times \frac{4}{3} \times \frac{3}{5} = \frac{12}{30}$
7.	$(-3) + (-5) = -8$
8.	$2^3 = 8$
9.	$\sqrt{14} = 3.742$

B. The following mathematical problems include their step-by-step solution procedures.

Explain the procedure to solve each of the mathematical problems by describing the sequence of steps.

1.	$(6 \times 4) \div 12 + 72 \div 8 - 9 =$ $(24 \div 12) + (72 \div 8) - 9 =$ $2 + 9 - 9 =$ $11 - 9 =$ 2
2.	$5x - 15 = 3x - 1$ $\quad +15 \quad +15$ $5x = 3x + 14$ $\quad -3x \quad -3x$ $2x = 14$ $\frac{2x}{2} = \frac{14}{2}$ $x = 7$
3.	$V = 60$

$$r = 3$$

$$V = \pi r^2 h$$

$$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

$$h = \frac{V}{\pi r^2}$$

$$h = \frac{60}{9\pi}$$

C. What mathematics topic has been the most difficult to learn and to teach? Why?

Appendix D



University of Costa Rica

Maestría Profesional en la Enseñanza del Inglés como Lengua Extranjera

Instructors: Marilyn Araya Cedeño, Pamela Fallas Brenes & Randall Montero Bolaños

Schedule: Tuesday 5:00 – 7:00 hrs

Modality: Virtual

Description

This course is designed with the purpose of providing students with sufficient input in the macro skills of listening, reading, and speaking. Likewise, it is intended to provide students with functional vocabulary, pronunciation, fluency, and structure as well as the micro skills related to mathematics teaching.

Goals and Objectives

Unit 1: Delivering Arithmetic, Algebra, and Geometry Input

Goal: At the end of the unit, students will be able to develop teaching materials for basic arithmetic, algebra, and geometry lessons by identifying and using key vocabulary and expressions from authentic listening and reading texts in the target language.

General Objectives: By the end of the lesson, students will be able to:

3. correctly show understanding of basic vocabulary and expressions related to arithmetic, algebra, and geometry by compiling a glossary videos of authentic mathematics lessons.

4. correctly show understanding of basic vocabulary and expressions related to arithmetic, algebra, and geometry by completing reading and listening exercises about explanations and descriptions of mathematical operations and procedures.
4. correctly apply a simple teaching resource to explain the pronunciation of basic vocabulary and expressions related to arithmetic, algebra, and geometry by using digital tool for listening, repeating, and reading aloud.

Unit 2: Delivering Arithmetic, Algebra, and Geometry Lessons

Goal: At the end of the unit, students will be able to explain basic mathematical procedures related to arithmetic, algebra, and geometry by using the correct vocabulary and expressions, modal verbs, sequencing markers, and intelligible pronunciation.

General Objectives: By the end of the lesson, students will be able to:

5. successfully identify target vocabulary necessary for arithmetic, algebra, and geometry lessons by analyzing written and audio passages containing explanations of mathematical procedures.
6. accurately organize the steps of mathematical procedures in the correct sequence in spoken form by using sequencing words and modal verbs.
7. accurately describe mathematical procedures orally by using previously learned vocabulary, appropriate sequencing words and modal verbs, and intelligible pronunciation.
8. effectively explain mathematical procedures related to arithmetic, algebra, and geometry by simulating a mathematics teacher describing procedures in an oral presentation in class.

Unit 3: Delivering Basic Geometry and Trigonometry Lessons

Goal: At the end of the unit, students will be able to adequately deliver basic geometry and trigonometry lessons by explaining introductory level examples and problems using the corresponding vocabulary and sequencing markers.

General Objectives: By the end of the lesson, students will be able to:

4. effectively identify basic vocabulary and sequencing markers necessary for geometry lessons by completing listening and reading exercises from authentic texts.
5. effectively identify and use basic vocabulary and sequencing markers necessary for basic trigonometry lessons by completing listening and reading exercises from authentic texts.
6. successfully deliver basic geometry and trigonometry lessons by using the correct vocabulary, sequencing markers, and modal verbs.

Methodology

This course allows students to integrate reading, listening, and speaking into their academic, professional, and real-life needs. Those needs are reflected in different tasks developed in three units. Lessons focused on communication and meaning, allowing students to clarify information with their classmates and instructors, recognize and apply technical vocabulary in different contexts, learn the pronunciation of new words, comprehend texts related to basic mathematical concepts, and develop successful listening skills in mathematics instruction. Instructors use authentic material such as texts, audios, worksheets, and pictures.

An active role is expected from students as by participating in communicative tasks in pairs or groups, asking questions, and clarifying or rephrasing information. As part of the students' participation in the course, they are expected to attend classes on a regular basis.

Course Evaluation

Course Evaluation

Assessment	Value
Listening comprehension quizzes (3)	15%
Reading comprehension quizzes (3)	15%
Course glossary	10%
Attendance	10%
Final Oral Exam	25%
Term Oral Project	25%
Total	100%

Contents

- Unit 1: Delivering Arithmetic, Algebra, and Geometry Input
- Unit 2: Delivering Arithmetic, Algebra, and Geometry Lessons
- Unit 3: Delivering Basic Geometry and Trigonometry Lessons

Appendix E

Class Preparedness, Participation and Engagement

Universidad de Costa Rica
 Escuela de Lenguas Modernas
 Maestría Profesional en la Enseñanza del Inglés
 PF-0311 Diseño de Práctica Profesional
 Limit Breakers English Club

Read the following statements and self-evaluate your class preparedness, participation, and engagement.

1. I am ready to start the class on time.
 1. Usually
 2. Sometimes
 3. Hardly never
 2. I come to class prepared. I do all course tasks and activities assigned as homework.
 1. Usually
 2. Sometimes
 3. Hardly never
 3. I have my camera on during class.
 1. Usually
 2. Sometimes
 3. Hardly never
 4. I participate and make contributions to class and group activities.
 1. Usually
 2. Sometimes
 3. Hardly never
 5. I volunteer to answer the teacher's questions.
 1. Usually
 2. Sometimes
 3. Hardly never
 6. How would you rate your overall class preparedness, participation, and engagement?
 - 1 (deficient)
 - 2
 - 3
 - 4
 - 5 (outsanding)
 7. What do you think about this course?
-

8. What do you like or enjoy most about this course?

9. How can we improve this course?

Thank you!

Appendix F

Mixed-Levels Strategy Input
Universidad de Costa Rica
Escuela de Lenguas Modernas
Maestría Profesional en la Enseñanza del Inglés
PF-0311 Diseño de Práctica Profesional
Limit Breakers English Club

¿Cual es su nombre completo? _____

1. ¿Que nivel de inglés considera que tiene?

- a. Básico
- b. Intermedio
- c. Avanzado

2. ¿Como se sintió al trabajar con compañeros que tenían su mismo nivel de inglés o similar? Especialmente cuando trabajaron en grupos pequeños o parejas.

3. ¿Como se sintió al trabajar con compañeros que tenían un nivel de inglés diferente al suyo, ya sea más bajo o más alto? Especialmente cuando trabajaron en grupos pequeños o parejas.

4. ¿Que considera es más beneficioso para usted, trabajar con compañeros que poseen un nivel de inglés similar al suyo o trabajar con compañeros que poseen un nivel diferente al suyo? Especialmente cuando trabajaron en grupos pequeños o parejas. Explique su respuesta

5. ¿Como se sintió al tener guías de trabajo individualizadas de acuerdo con su nivel de inglés? Específicamente cuando trabajaron en grupos pequeños o parejas.

6. ¿Considera beneficioso el uso de las guías de trabajo individualizadas de acuerdo con su nivel de inglés? Explique su respuesta.

Appendix G

Final Oral Exam

Read aloud the following solution procedures, step by step, for basic mathematical operations and equations.

Remember to use sequencing language like 'first', 'then', 'next', 'finally'. Pay attention to your own pronunciation.

Order of Operations

$$\begin{aligned} &(3^2 \times 4) + 6 \\ &(3^2 \times 4) + 6 \\ &(9 \times 4) \\ &36 + 6 \\ &42 \end{aligned}$$

$$\begin{aligned} &10 \times (4 + 5) \\ &10 \times (4 + 5) \\ &\quad (9) \\ &10 \times 9 \\ &90 \end{aligned}$$

Basic Equations

$$\begin{aligned} x + 7 &= 15 \\ -7 \quad -7 \\ \hline x &= 8 \end{aligned}$$

$$\begin{array}{r} 40 = x + 25 \\ -25 \quad -25 \\ \hline 15 = x \\ \text{x} = 15 \end{array}$$

$$\begin{array}{r} 12x = 96 \\ \hline \cancel{12}x = \cancel{12} \cdot 8 \\ \hline x = 8 \end{array}$$

$$\begin{array}{r} 3x = 15 \\ \hline \cancel{3}x = \cancel{3} \cdot 5 \\ \hline x = 5 \end{array}$$

$$\frac{x}{10} = 15$$

$$(10)\frac{x}{10} = 15(10)$$

$$\cancel{(10)}\frac{x}{\cancel{10}} = 15(10)$$

$$x = 150$$

Step Equations

$$2x + 2 = 8$$

$$2x + 2 = 8$$

$$\quad -2 \quad -2$$

$$\cancel{2}x = \underline{6}$$

$$\quad \cancel{2} \quad 2$$

$$x = 3$$

$$\frac{x}{2} - 1 = 4$$

$$\frac{x}{2} - 1 = 4$$

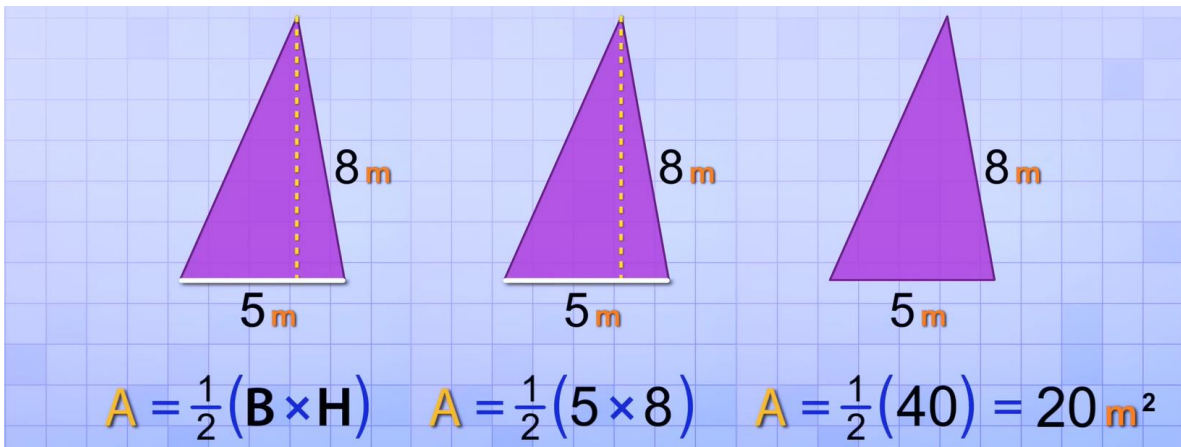
$$\quad +1 \quad +1$$

$$\cancel{(2)}\frac{x}{\cancel{2}} = 5(2)$$

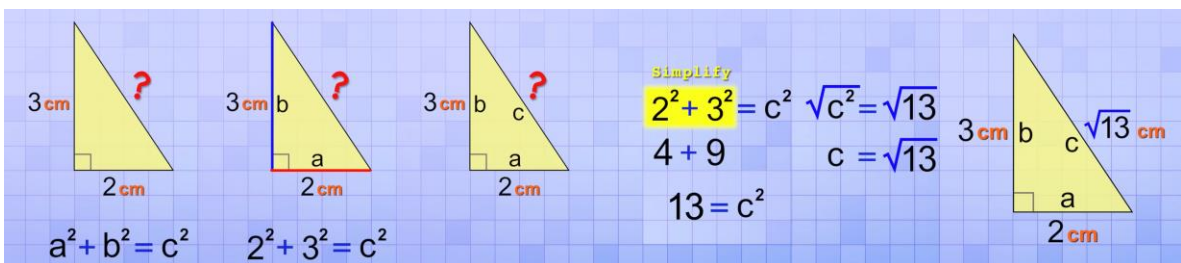
$$x = 10$$

$$\begin{aligned}
 2(x+2) &= 8 \\
 \cancel{2}(x+2) &= \frac{8}{\cancel{2}} \\
 (x+2) &= 4 \\
 x+2 &= 4 \\
 -2 &-2 \\
 x &= 2
 \end{aligned}$$

Triangles



The Pythagorean Theorem



Appendix H

Lesson Plan – Week 1

University of Costa Rica

Date: 8/15/2023

Master’s Program in TEFL

Student teacher: The Entire Team

Limit Breakers English Club: An Online English Course for Mathematic Students

Assistant/s: The Entire Team

Lesson Plan # 1



Introduction of Unit # 1 Title of unit: Delivering Arithmetic, Algebra, and Geometry Input

Unit Goal: By the end of the unit, students will be able to show understanding of key vocabulary and common expressions related to arithmetic, algebra, and geometry by listening to and reading authentic input in the target language.

Specific Objectives: By the end of this lesson, students will be able to...

1. introduce themselves using basic personal information.
2. successfully explain their answers to algebra-related questions by writing and reading aloud their ideas.
3. accurately use target vocabulary when discussing algebra-related topics.
4. effectively collaborate in small groups to answer algebra-related questions.

Objectives	Procedures	Language	Strategies	Macro	Time
------------	------------	----------	------------	-------	------

				Skills	
	<p>Introduction to the Course</p> <p>T welcomes everyone and introduces the course.</p>			L	10 min
	<p>Student introductions</p> <p>Ss introduce themselves by saying their name, where they are from, and one of their hobbies or interests. Ts provide feedback as needed.</p>	<ul style="list-style-type: none"> ● Hi! / Hello! / Good Evening! ● My name is _____. ● I am from_____. ● My favorite hobby is_____ / I am interested in_____. ● My perfect day would start with _____ and end with_____. 	Modeling	S	20 min
	<p>Warm - up activity</p> <p>The Ss complete useful expressions with facts about themselves. Then, every S shares one of their facts with the class.</p>	<ul style="list-style-type: none"> ● I chose number ____ because it is_____. ● My numerical fact is _____ because it represents_____. 	Modeling Personalizing	S	20 min

	<p>Jamboard link:</p> <p>https://jamboard.google.com/d/1GXoH6A8WRWcF_1PHWKNT-NV7lwsfTWc658337e5HXc8/edit?usp=sharing</p>				
	<p>Course outline</p> <p>T reads and explains course outline including objectives, activities, methodology, evaluation, and resources.</p>	<ul style="list-style-type: none"> ● I have a question. ● Would you please repeat that? ● I did not understand that part. <p>Would you mind explaining that again?</p> <ul style="list-style-type: none"> ● I get it now, thank you! 	<p>Visual aids</p> <p>Clarification</p>	<p>R, L</p>	<p>20 min</p>
	<p>Course platforms</p> <p>T introduces Google Classroom, Drive, and Zoom and explains how to access them.</p>	<ul style="list-style-type: none"> ● I have a question. ● Would you please repeat that? ● I did not understand that part. <p>Would you mind explaining that</p>	<p>Visual aids</p> <p>Asking for clarification</p>	<p>R, L</p>	<p>20 min</p>

		<p>again?</p> <ul style="list-style-type: none"> • I get it now, thank you! 			
1	<p>Warm-up for Unit 1: Class is divided into groups of three students. Ss use the platform Padlet to answer three algebra-related questions in a collaborative manner. When they return to the main room in Zoom, they share and discuss their answers. The questions are the following:</p> <ol style="list-style-type: none"> 1. What is algebra? 2. What is an equation? 3. Why is algebra important? In what fields is algebra typically used? <p>Feedback: ATs and T encourage</p>	<p>Procedural:</p> <ul style="list-style-type: none"> -What is the word for...? -What is the pronunciation of...? -What do you think...? -In my opinion... -I think... -I'm not sure about this... -I agree/disagree <p>Vocabulary:</p> <ul style="list-style-type: none"> • Algebra is... • An equation is... / The definition of an equation is... • Algebra is important because... • Algebra is typically used in the 	<p>Schema activation</p> <p>Small group sharing</p>	<p>R</p> <p>W</p> <p>S</p>	15 min

	<p>participation and monitor Ss' answers.</p> <p>Materials: Padlet</p> <p>https://padlet.com/maraya295/algebra-28iufieo4wjmcz0z</p>	<p>fields of...</p> <p>Numbers, negative numbers.</p> <p>Verbs: add, subtract, divide by, multiply by, simplify.</p> <p>Nouns: addition, subtraction, multiplication, division, dividend, divisor, quotient, remainder, product, numerator, denominator, like/unlike fractions, decimals.</p>			
--	--	---	--	--	--

Abbreviations to be used: T = Teacher

ATs = Assistant Teachers

Ss = students

BORs = Breakout

Rooms

Lesson Plan – Week 2



University of Costa Rica

Master's Program in TEFL

Limit Breakers English Club: An Online English Course for Mathematics Students

Lesson Plan # 2

Date: 8/22/2023

Student teacher: Randall Montero

Assistants: Pamela Fallas & Marilyn Araya

Unit # 1: Delivering Arithmetic, Algebra, and Geometry Input

Unit Goal: At the end of the unit, students will be able to develop teaching materials for basic arithmetic, algebra, and geometry lessons by identifying and using key vocabulary and expressions from authentic listening and reading texts in the target language.

General Objective: By the end of the lesson, students will be able to correctly show understanding of basic vocabulary and expressions related to arithmetic, algebra, and geometry by completing reading and listening exercises based on explanations and descriptions of mathematical operations and procedures.

Specific Objectives: By the end of this lesson, students will be able to...

1. effectively activate their background knowledge of algebra and equations by answering basic related questions and discussing the answers while interacting with classmates.

2. effectively recognize vocabulary words related to algebra and arithmetic by playing a memory game.
3. accurately identify vocabulary words about algebra and arithmetic by watching an explanatory video and completing a handout.
4. accurately show understanding of vocabulary related to algebra by matching the word with its corresponding definition.
5. effectively explain algebra basics in a role play between a math teacher and a student by using previously learned vocabulary to complete the role-play script.
6. accurately articulate the sounds “shun” [ʃ] and “zhun” [ʒ] by recording themselves reading a list of words aloud.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
1	<p>Warm-up:</p> <p>As a group, Ss use two algebra-related questions on the Padlet platform as a prompt to provide keywords simultaneously (whole class). Then, Ss read the keywords</p>	<p>To talk to classmates:</p> <p>What is the word for...?</p> <p>What is the pronunciation of...?</p> <p>What do you think...?</p> <p>In my opinion...</p> <p>I think...</p>	<p>Schema activation</p> <p>Group sharing</p>	<p>R</p> <p>W</p> <p>S</p>	<p>10 min</p>

	<p>aloud and T provides feedback as needed.</p> <p>The questions are the following:</p> <ol style="list-style-type: none"> 1. What is algebra? 2. What is an equation? <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Padlet</p> <p>https://padlet.com/maraya295/algebra-28iufieo4wjmcz0z</p>	<p>I am not sure about this...</p> <p>I agree / I disagree</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>Vocabulary:</p> <p>Numbers, negative numbers.</p> <p>Verbs: add, subtract, divide by, multiply by, simplify.</p> <p>Nouns: addition, subtraction, multiplication, division, dividend, divisor, quotient, remainder, product, numerator, denominator, like/unlike fractions,</p>			
--	---	---	--	--	--

		decimals.			
2	<p>Pre-task 1:</p> <p>As a group, Ss play a memory game to introduce and recognize useful vocabulary related to algebra and arithmetic.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: online memory game https://interacty.me/projects/981b4f976b5149c7</p>	<p>To talk to classmates:</p> <p>I would say that the number is...</p> <p>I choose number... and number...</p> <p>I think it is number...</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>I get it now, thank you!</p> <p>Vocabulary:</p> <p>Numbers, negative numbers.</p> <p>Verbs: add, subtract, divide by, multiply by, simplify.</p>	<p>Identifying vocabulary</p> <p>Group sharing</p>	S	20 min

		Nouns: addition, subtraction, multiplication, division, dividend, divisor, quotient, remainder, product, numerator, denominator, like/unlike fractions, decimals.			
3	<p>Pre-task 2:</p> <p>Ss watch the first 3 min of a video explaining what algebra is with examples. They pay attention to the use of vocabulary and demonstrate comprehension by completing a filling in the blanks exercise individually.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 1</p>	<p>To talk to classmates:</p> <p>What is the word for...?</p> <p>What is the pronunciation of...?</p> <p>I am not sure about this...</p> <p>How do you say...?</p> <p>I think it is...</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p>	<p>Listening to vocabulary words and phrases</p> <p>Recognizing key vocabulary</p> <p>Matching symbols with examples</p> <p>Group sharing</p>	L W	20 min

	<p>Video:</p> <p>https://www.youtube.com/watch?v=NybHckSEQBI</p>	<p>Vocabulary:</p> <p>Numbers, negative numbers.</p> <p>Verbs: add, subtract, divide by, multiply by, simplify.</p> <p>Nouns: addition, subtraction, multiplication, division, dividend, divisor, quotient, remainder, product, numerator, denominator, like/unlike fractions, decimals.</p>			
4	<p>Pre-task 3:</p> <p>Working in pairs, Ss match mathematical vocabulary (9 items) with their definitions using handout 2.</p> <p>T and ATs model the instructions for students, answer questions, and</p>	<p>To talk to classmates:</p> <p>What do you think?</p> <p>I am not sure...</p> <p>I think it is ... / I don't think it is ...</p> <p>I agree / I disagree</p> <p>What about...?</p>	<p>Identifying vocabulary and definitions.</p> <p>Pair work</p>	R	20 min

	<p>provide feedback as needed.</p> <p>After 10 min doing the activity, T reviews answers with the class by asking Ss to share their answers.</p> <p>Materials: Handout 2</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>Vocabulary:</p> <p>Numbers, negative numbers.</p> <p>Verbs: add, subtract, divide, multiply, simplify.</p> <p>Nouns: addition, subtraction, multiplication, division, variable, algebraic expression, quotient, remainder, product, etc.</p>			
5	<p>Task:</p> <p>Working in pairs, Ss choose a role in a</p>	<p>To talk to classmates:</p> <p>What do you think?</p>	Pair Work	R W	30 min

	<p>dialogue. Student A plays the role of a math teacher while student B plays the role of the student. Using handout 3, Ss complete the conversation by filling in the blanks. After that, they role-play the conversation a few times. Finally, pairs work in BORs with a T or an ATs to carry out the role-play and receive feedback.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 3</p>	<p>I am not sure...</p> <p>I think it is ... / I don't think it is ...</p> <p>I agree / I disagree</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>Vocabulary:</p> <p>Numbers, negative numbers.</p> <p>Verbs: add, subtract, divide, multiply, simplify.</p> <p>Nouns: addition, subtraction, multiplication, division, variable, algebraic expression, quotient, remainder, product,</p>	<p>Small group sharing</p> <p>Semi-structured role-play</p>	<p>S</p>	
--	--	---	---	----------	--

		etc.			
6	<p>Post-task:</p> <p>Asynchronously, Ss watch 3 videos about the pronunciation of the sounds “shun” [ʃ] & “zhun” [ʒ]. After watching the video, Ss practice a list of words and get ready to pronounce them correctly the next class.</p> <p>Feedback:</p> <p>In the next class, ATs and T listen to the Ss pronouncing the list of words and provide feedback as needed.</p> <p>Materials: Handout 4</p> <p>Video #1: https://www.youtube.com/watch?v=RxaLKZPPEvY</p>	<p>Consonant sounds “shun” [ʃ] as in the word <i>operation</i> and “zhun” [ʒ] as in the word <i>equation</i></p>	Pronunciation	S	60 min

	Video #2: https://www.youtube.com/watch?v=wI-Nb4HFguck&t=604s Video #3: https://www.youtube.com/watch?v=k8ImSmVOSVA				
--	--	--	--	--	--

Abbreviations used: T = Teacher ATs = Assistant Teachers Ss = students min = minutes BORs = breakout rooms

Lesson Plan – Week 3

University of Costa Rica

Date: 8/29/2023

Lesson Plan # 3

Master's Program in TEFL

Student teacher: Pamela Fallas

Limit Breakers English Club: An Online English Course for Mathematics Students

Assistants: Randall Montero & Marilyn Araya



Unit # 1: Delivering Arithmetic, Algebra, and Geometry Input

Unit Goal: At the end of the unit, students will be able to develop teaching materials for basic arithmetic, algebra, and geometry lessons by identifying and using key vocabulary and expressions from authentic listening and reading texts in the target language.

General Objective: By the end of the lesson, show understanding of basic vocabulary and expressions related to arithmetic and algebra by completing reading and listening exercises based on explanations and descriptions of mathematical operations and procedures.

Specific Objectives: By the end of this lesson, students will be able to...

1. activate their background knowledge of algebra and equations by reviewing previously learned vocabulary.
2. effectively recognize words related to algebra and arithmetic by solving a multiple-choice exercise.
3. accurately identify vocabulary words about algebra and arithmetic by listening to an explanatory recording and completing a handout.
4. effectively explain the order of operations by using previously learned vocabulary to organize a conversation in logical order.

5. accurately explain a procedure by using the order of operations and sequencing words.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
1	<p>Welcome Class</p> <p>Warm-up: Ss volunteer to read aloud the new terms they learned in the previous lesson (week 2), and T provides feedback as needed.</p> <p>ATs and T model the instructions for Ss, answer questions, and provide feedback as needed.</p> <p>Materials: (Activity 1)</p>	<p>To talk to classmates:</p> <p>What is the word for...?</p> <p>What is the pronunciation of...?</p> <p>What do you think...?</p> <p>In my opinion...</p> <p>I think...</p> <p>I am not sure about this...</p> <p>I agree/ I disagree</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p>	Schema activation	R S	10 min

		<p>Vocabulary:</p> <p>Multiplication, Addition, Subtraction, Operation, Pronunciation, Fraction, Attention, Definition, Quotient, Ratio, Vision, Division, Equation, Decision, Conversion, Precision, Conclusion, Revision.</p>			
<p>2</p>	<p>Pre-task 1: Ss work individually and use the platform Quizizz to review vocabulary words and expressions related to arithmetic by solving a multiple-choice exercise. After that, Ss and T check answers together.</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>I get it now, thank you!</p> <p>Vocabulary:</p> <p>Numbers, negative numbers.</p>	<p>Identifying vocabulary</p> <p>Group sharing</p>	<p>S</p>	<p>20 min</p>

	<p>ATs and T model the instructions for Ss, answer questions, and provide feedback as needed.</p> <p>Materials: Quizizz</p> <p>https://quizizz.com/admin/quiz/64eb6f682e25b2ce78b1ba78?source=quiz_share</p>	<p>Verbs: add, subtract, divide by, multiply by, simplify.</p> <p>Nouns: addition, subtraction, multiplication, division, dividend, divisor, quotient, remainder, product, numerator, denominator, like/unlike fractions, decimals, brackets, braces, exponents, power, index.</p>			
3	<p>Pre-task 2:</p> <p>Ss listen to a recording about the order of operations in arithmetic to complete a fill in the blanks exercise.</p> <p>ATs and T model the instructions for Ss, answer questions, and provide feedback as needed.</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>Vocabulary:</p> <p>Numbers, negative numbers.</p>	<p>Active listening</p> <p>Questioning for clarification</p>	L	20 min

	Materials: Handout 1, self-recording	Verbs: add, subtract, divide by, multiply by, simplify. Nouns: addition, subtraction, multiplication, division, dividend, divisor, quotient, remainder, product, numerator, denominator, like/unlike fractions, decimals,			
4	Task: Ss work in pairs and choose a role. Student B is struggling to solve a math problem while student A is helping student B to solve it. Using handout 3, Ss organize the conversation in logical order. After that, they role-play the conversation a few times. Finally, 3 pairs are grouped together in a breakout room	To talk to classmates: What do you think? Do you know what _____ means? I am not sure... I think the _____ (color) strip goes before/after the _____ (color) strip. I agree... I disagree Colors: purple, gray, brown, dark green, light green, orange, blue, yellow, pink,	Pair Work Small group sharing Semi-structured role-play	S	35 min

	<p>with a T or ATs to carry out the role-play.</p> <p>ATs and T model the instructions for Ss, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 2</p>	<p>red.</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>Vocabulary:</p> <p>Exponents, base, symbols, left, right, parenthesis, braces, brackets.</p> <p>Verbs: add, subtract, divide by, multiply by, simplify.</p>			
<p>5</p>	<p>Post-task:</p> <p>Asynchronously, Ss complete handout 3 about solving problems using order of operations and sequencing language.</p>	<p>Sequencing markers: first, next, then, after that, last, finally, before, after</p>	<p>Sequencing</p>	<p>W S</p>	<p>15 min</p>

	<p>Feedback:</p> <p>In the next class, ATs and T listen to the Ss read aloud their problems step by step using sequencing markers and provide feedback as needed.</p> <p>Materials: Handout 3</p>				
--	--	--	--	--	--

Abbreviations to be used: T = Teacher

ATs = Assistant Teachers

Ss = students

min = minutes

Lesson Plan – Week 4

University of Costa Rica

Date: 9/05/2023

Lesson Plan # 4

Master's Program in TEFL

Student teacher: Marilyn Araya

Limit Breakers English Club: An Online English Course for Mathematics Students

Assistants: Randall Montero & Pamela Fallas



Unit # 1: Delivering Arithmetic, Algebra, and Geometry Input

Unit Goal: At the end of the unit, students will be able to develop teaching materials basic for arithmetic, algebra, and geometry lessons by identifying and using key vocabulary and expressions from authentic listening and reading texts in the target language.

General Objective: By the end of the lesson, successfully recognize basic vocabulary and expressions related to geometry by completing reading and listening exercises based on explanations and descriptions of mathematical operations and procedures.

Specific Objectives: By the end of this lesson, students will be able to...

1. effectively name geometric shapes by reading some characteristics about them.
2. effectively locate geometry terms and definitions by color-coding them in the text.
3. successfully complete written captions by listening to short recordings.
4. effectively complete a conversation about geometry by using previously learned vocabulary words and expressions in a role-play script.
5. successfully identify vocabulary words and their characteristics by playing a drawing game.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
-------------------	-------------------	-----------------	-------------------	---------------------	-------------

<p>1</p>	<p>T welcomes the class and asks for volunteers to share their homework.</p> <p>Warm-up: T reads a description of some geometric shapes. Ss volunteer to choose the shape that belongs to the description provided.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Activity 1</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>To participate in the activity:</p> <p>I think it is a...</p> <p>The answer is...</p> <p>I am not sure...</p> <p>Vocabulary:</p> <p>Square, triangle, hexagon, pentagon, cube, rectangle, heptagon, octagon, nonagon, cone, cylinder, pyramid, circle</p>		<p>S</p> <p>L</p>	<p>10 min</p>
----------	---	---	--	-------------------	---------------

2	<p>Pre-task 1:</p> <p>Students work in groups of three. Ss read a text about geometry and color-code geometry terms and definitions.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 1</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>I get it now, thank you!</p> <p>To talk to classmates:</p> <p>How do you say this word?</p> <p>What do you think...?</p> <p>I think...</p> <p>I am not sure about this...</p> <p>I agree/ I disagree</p> <p>Let's highlight it _____ (color).</p> <p>It is here on the first/second/third paragraph.</p> <p>Vocabulary:</p>		R	20 min
---	--	---	--	---	--------

		Line, line segment, ray, plane, point, 2d/ 3d shapes, circle, triangle, square, rectangle, sphere, cube, cone, cylinder.			
--	--	--	--	--	--

3	<p>Pre-task 2:</p> <p>Individually, Ss complete written captions by listening to short recordings.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 2</p>	<p>To participate in the activity:</p> <p>I can try!</p> <p>I'll do it!</p> <p>I have a question.</p> <p>I don't know how to pronounce this word.</p> <p>I think it is_____.</p> <p>Vocabulary: pot, geometry, dot, location, line, endpoints, line segment, shape, ray, infinity, plane, flat surface, width, length.</p>		L	20 min
---	--	--	--	---	--------

4	<p>Task: Ss work in groups of three and choose a role. Student A and B are students while student C is a teacher. Using handout 3, Ss complete a conversation about geometry. After that, they role-play the conversation a few times. Finally, 3 pairs are grouped together in a breakout room with a T or ATs to carry out the role-play. ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 3</p>	<p>To talk to classmates:</p> <p>What do you think? Do you know what _____ means? We can say that... We can add that... I am not sure... I agree... I disagree</p> <p>To ask the teacher:</p> <p>I have a question. Would you please repeat that? I did not understand. Would you mind explaining again?</p> <p>Vocabulary:</p> <p>Square, triangle, hexagon, pentagon, cube, rectangle, heptagon, octagon, nonagon, cone, cylinder, pyramid, circle,</p>		S	35 min
---	--	---	--	---	--------

		lines, planes, points, edges.			
--	--	-------------------------------	--	--	--

5	<p>Post-task: Working in pairs and using the Jamboard platform, Ss take turns to describe and draw some geometric shapes, lines, points, and planes according to the vocabulary and definitions learned previously.</p> <p>Feedback:</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Jamboard, flashcards</p> <p>https://jamboard.google.com/d/1KPMX8zpsAsfoWbjYDUMK04XL4aG_HYqIGtUmcWNGbxY/edit?usp=sharing</p>	<p>To talk to classmates:</p> <p>It is my turn to draw/describe...</p> <p>Let me try...</p> <p>I think the description corresponds to...</p> <p>Is that a...?</p> <p>Could you repeat that?</p> <p>I did not understand. Would you mind saying that again?</p> <p>I would say that the answer is...</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>Vocabulary:</p> <p>Square, triangle, hexagon, pentagon,</p>		L S	10 min
---	---	---	--	--------	--------

		cube, rectangle, heptagon, octagon, nonagon, cone, cylinder, pyramid, circle, lines, planes, points, edges.			
--	--	---	--	--	--

6	<p>Asynchronous activity: T provides a list of vocabulary words related to geometry. Ss practice the correct pronunciation of each of them and record themselves. They send this recording to their T.</p> <p>Feedback: Ss receive feedback as needed in terms of pronunciation.</p> <p>Materials: Handout 4</p>	<p>Vocabulary:</p> <p>Square, triangle, hexagon, pentagon, cube, rectangle, heptagon, octagon, nonagon, cone, cylinder, pyramid, circle, lines, planes, points, edges</p>		S	60 min
---	---	--	--	---	--------

Abbreviations to be used: T = Teacher

ATs = Assistant Teachers

Ss = students

min = minutes

Lesson Plan – Week 5
University of Costa Rica



Master's Program in TEFL

Lesson Plan # 5

Date: 9/12/2023

Student teacher: Randall Montero

Limit Breakers English Club: An Online English Course for Mathematics Students

Assistants: Marilyn Araya & Pamela Fallas

Unit # 1: Delivering Arithmetic, Algebra, and Geometry Lessons

Unit Goal: At the end of the unit, students will be able to explain basic mathematical procedures related to arithmetic, algebra, and geometry by using the correct vocabulary and expressions, modal verbs, sequencing markers, and intelligible pronunciation.

General Objective: By the end of the lesson, students will be able to identify vocabulary necessary for basic arithmetic, algebra, and geometry lessons by analyzing written and audio passages containing explanations of mathematical expressions and procedures.

Specific Objectives: By the end of this lesson, students will be able to...

1. validate vocabulary related to algebra and geometry studied in the previous class.
2. determine vocabulary related to geometry by formulating and solving area formulas.
3. determine vocabulary related to arithmetic and algebra by reading mathematical expressions correctly.

4. demonstrate understanding of vocabulary related arithmetic, algebra, and geometry by completing and role-playing a dialog.
5. determine the correct pronunciation of vocabulary related arithmetic, algebra, and geometry by completing a pronunciation exercise.

Lesson Plan

Objectives	Procedures	Language	Strategies	Macro Skills	Time
------------	------------	----------	------------	--------------	------

<p>1</p>	<p>T welcomes the class and asks for early attendees to share how their weekend was by using an emoji from the Reactions menu.</p> <p>Warm-up:</p> <p>As a class, Ss do a matching exercise with the T. In this exercise, Ss match mathematical terms previously studied in class with their respective branch (algebra, and geometry).</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Activity 1</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>I get it now. Thank you!</p> <p>To participate in the activity:</p> <p>I believe [<i>word</i>] goes with [<i>branch</i>].</p> <p>I am not sure, but I think [<i>word</i>] goes with [<i>branch</i>].</p> <p>Vocabulary:</p> <p>addition, subtraction, division, multiplication, equation, value, algebraic expression, exponents, placeholder, lines, angles, points, planes, shapes, square, circle, triangle, cone</p>		<p>S</p>	<p>10 min</p>
----------	---	--	--	----------	---------------

<p>2</p>	<p>Pre-task 1:</p> <p>Individually, Ss watch a video on how to mathematically calculate the area of squares, rectangles, and triangles. Then, in groups of three, Ss answer four questions related to formulas and equations using what they learned from the video while helping and giving feedback to one another. After five minutes in the BORs answering the questions, Ss watch two other videos to check their answers. ATs and T read the handout questions, model the instructions for students, answer any questions Ss have, and provide feedback as</p>	<p>To participate in the activity:</p> <p>I want to answer question one / two / three / four.</p> <p>I can answer question one / two / three / four.</p> <p>I don't know how to say [<i>Spanish word</i>] in English.</p> <p>I don't remember how to say [<i>Spanish word</i>] in English.</p> <p>I forgot how to say [<i>Spanish word</i>] in English.</p> <p>How do you say [<i>Spanish word</i>] in English?</p> <p>I think the answer is _____.</p> <p>I think in English you say...</p> <p>In Spanish, we say..., but in English, they</p>		<p>L</p> <p>S</p>	<p>20 min</p>
-----------------	---	--	--	-------------------	---------------

	<p>needed.</p> <p>Materials: Handout 1; Pre-task 1 videos 1, 2, and 3.</p>	<p>say...</p> <p>Vocabulary: base, height, length, width, square, rectangle, triangle, acute, obtuse.</p>			
--	---	--	--	--	--

<p>3</p>	<p>Pre-task 2:</p> <p>Individually, Ss watch a video on how to read basic mathematical expressions. Then, the teacher goes over handout 2 (guide on how to read basic mathematical expressions) with the class. After that, in groups of three (BORs), Ss read aloud the equation in handout 3 using the vocabulary and tips provided in the video and handout 2 while helping and giving feedback to one another.</p> <p>After five minutes in the BORs reading and writing down the equation in handout 3, Ss watch another video to check their answers. T asks for</p>	<p>To participate in the activity:</p> <p>The first part of the equation is...</p> <p>The next part of the equation is...</p> <p>The last part of the equation is...</p> <p>I don't know how to say [Spanish word] in English.</p> <p>I don't remember how to say [Spanish word] in English.</p> <p>I forgot how to say [Spanish word] in English.</p> <p>How do you say [Spanish word] in English?</p> <p>I think the answer is _____.</p> <p>I think the word in English is _____.</p> <p>I think in English you say...</p> <p>In Spanish, we say..., but in English, they</p>		<p>L</p> <p>R</p> <p>S</p>	<p>20 min</p>
-----------------	---	---	--	----------------------------	---------------

	<p>volunteers to read the equation to check pronunciation.</p> <p>ATs and T model the instructions for students, answer questions and provide feedback as needed.</p> <p>Materials: Handouts 2 and 3; Pre-task 2 videos 4 and 5</p>	<p>say...</p> <p>Vocabulary: negative, minus, fraction, quarter, power, square, cube, root, less, greater, equal, plus, multiplied, divided.</p>			
--	--	---	--	--	--

4	<p>Task:</p> <p>Ss work in groups of three and choose a role. Student A is a teacher, Student B is a student and Student C is also a student. Using handout 4 Ss first complete a conversation about calculating areas and reading equations. After that, they role-play the conversation a few times switching roles. Finally, Ss are grouped together in BORs with a T or one AT to carry out the role-play. ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 4</p>	<p>To talk to classmates:</p> <p>What do you think?</p> <p>Do you know what ____ means?</p> <p>I don't remember how to say [Spanish word] in English.</p> <p>How do you say [Spanish word] in English?</p> <p>We can say that...</p> <p>We can add that...</p> <p>I am not sure.</p> <p>I agree... I disagree...</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>I need your help.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind</p>		S	35 min
---	---	---	--	---	--------

		<p>explaining again?</p> <p>Vocabulary:</p> <p>area, square, rectangle, triangle, calculate, find, length, width, height, base, fraction, power, exponent</p>			
--	--	--	--	--	--

5	<p>Post-task:</p> <p>Working in pairs, Ss take turns to pronounce and classify the list of words in handout 5 based on the pronunciation of the letter <i>u</i>.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 5</p>	<p>To talk to classmates:</p> <p>What do you think?</p> <p>Do you think it is column one or column two?</p> <p>I think it is column one / two.</p> <p>Are you sure it is column one / two?</p> <p>Do you know what _____ means?</p> <p>I don't remember how to pronounce [word] in English.</p> <p>How do you say [Spanish word] in English?</p> <p>I am not sure...</p> <p>I agree... I disagree...</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p>		S	15 min
---	--	--	--	---	--------

		<p>I did not understand. Would you mind explaining again?</p> <p>Vocabulary:</p> <p>include, number, multiplication, sum, root, plus, rule, acute, minus, obtuse, multiply, numerator, subtraction</p>			
--	--	---	--	--	--

<p>6</p>	<p>Asynchronous post-task:</p> <p>Ss complete handout 6 about the sounds / ʌ / and / u / in words spelled with the letter < u >.</p> <p>Feedback:</p> <p>In the next class, ATs and T listen to the Ss read aloud the words with the sounds / ʌ / and / u / and provide feedback as needed.</p> <p>Materials:</p> <p>Handout 6</p>	<p>Vocabulary:</p> <p>luggage, but, tube, Tuesday, sun, introduce, much, trunk, just, suitcase, suit, blue, uniform, soup, wonderful</p>		<p>S</p>	<p>30 min</p>
-----------------	---	---	--	----------	---------------

Abbreviations to be used: T = Teacher ATs = Assistant Teachers Ss = students BOR = breakout room

min = minutes

Lesson Plan – Week 6

University of Costa Rica

Date: 9/19/2023

Lesson Plan #6

Master's Program in TEFL

Student teacher: Pamela Fallas

Limit Breakers English Club: An Online English Course for Mathematics Students

Assistants: Randall Montero & Marilyn Araya



Unit # 1: Delivering Arithmetic, Algebra, and Geometry Input

Unit Goal: At the end of the unit, students will be able to develop teaching materials for basic arithmetic, algebra, and geometry lessons by identifying and using key vocabulary and expressions from authentic listening and reading texts in the target language.

1. **General Objective:** By the end of the lesson, students will be able to demonstrate understanding of basic vocabulary and expressions related to arithmetic, algebra, and geometry by completing reading and listening exercises based on explanations and descriptions of mathematical operations and procedures.

Specific Objectives: By the end of this lesson, students will be able to...

1. identify vocabulary words and expressions related to arithmetic, algebra, and geometry by completing a set of sentences in a multiple-choice game.
2. describe the steps of basic mathematical operations and basic elements of geometry such as shapes and lines by using previously studied vocabulary words and expressions related to arithmetic, algebra, and geometry.

3. demonstrate clear understanding of basic vocabulary and expressions related to arithmetic, algebra, and geometry by completing reading and listening quiz based on explanations and descriptions of mathematical operations and procedures.
4. demonstrate understanding of the rule for the vowel sounds / ^ / and / u / by sorting out words.
5. locate and define new vocabulary related to algebra by watching videos, solving a word search, and completing a vocabulary chart.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
1	<p>T welcomes the class and asks early attendees to share a movie or TV show recommendation.</p> <p>Warm-up: Ss play Kahoot individually to review some vocabulary and expressions related to arithmetic, algebra, and geometry. The purpose</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining that again?</p> <p>Vocabulary:</p> <p>point, plane, shapes, line segment, ray, triangle, hexagon, cube, parentheses, power, exponent, simplify.</p>		<p>S</p> <p>R</p>	<p>5 min</p> <p>10min</p>

	<p>of this review is to help them to get ready for the quiz.</p> <p>AT and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Kahoot</p> <p>https://create.kahoot.it/details/f6fcadc4-af12-4aca-bf22-104a7e28519a</p>				
<p>2</p>	<p>Pre-task: In pairs, Ss take turns to describe the steps to solve problems using the order of operations correctly. Besides, Ss take turns to</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining that again?</p> <p>To participate in the activity:</p>		<p>S</p>	<p>15min</p>

	<p>describe some basic geometry elements such as shapes and lines.</p> <p>AT and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 1</p>	<p>I can try!</p> <p>I'll do it!</p> <p>I have a question.</p> <p>I don't know how to pronounce this word.</p> <p>I think it is_____.</p> <p>I am not sure.</p> <p>Vocabulary:</p> <p>first, next, then, after that, last, finally, before, after, square, triangle, hexagon, pentagon, cube, ray, plane, endpoint, infinity, sides, angles, flat.</p>			
<p>3</p>	<p>Task: T provides instructions for reading comprehension quiz #1.</p> <p>Students complete reading comprehension quiz #1. T provides</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p>		<p>R</p> <p>L</p>	<p>35min</p>

	<p>instructions for listening comprehension quiz #1. Students complete listening comprehension quiz #1.</p> <p>Materials: Listening quiz one Reading quiz one</p>				
4	<p>Post-task: T asks for volunteers to share their homework and provides feedback as needed.</p> <p>Materials: Handouts 5 Handout 6</p>	<p>Vocabulary: include, number, multiplication, sum, root, plus, rule, acute, minus, obtuse, multiply, numerator, subtraction.</p> <p>To participate in the activity: I can try! I'll do it! I have a question. I don't know how to pronounce this word.</p>		S	15min

		I think it is_____.			
		I am not sure.			
5	<p>Asynchronous Session:</p> <p>As part of preparation for the next class, students watch two videos about Algebra Basics. After watching the videos, Ss complete a word search to practice new vocabulary from the videos. As well as completing a chart by including the definition and pronunciation reference of the new vocabulary from the video.</p> <p>Materials: Handout 2</p> <p>Video 1: https://www.youtube.com/watch?v=l3XzepN03KQ&list=PL4mRaHZim4UQ</p>			L	30 min

	<p>PTOTHTe6uKnRoGSo5GuP4&index=2</p> <p>Video 2:</p> <p>https://www.youtube.com/watch?v=Qyd_v3DGzTM&list=PL4mRaHZim4UQPTOTHTe6uKnRoGSo5GuP4&index=3</p> <p>Word search:</p> <p>https://www.educima.com/wordsearches/vocabulary-87141f8091989bb54e50a7168c7628b9</p>				
--	---	--	--	--	--

Abbreviations to be used: T = Teacher

ATs = Assistant Teachers

Ss = students

min = minutes

Lesson Plan – Week 7

University of Costa Rica

Date: 9/26/2023

Lesson Plan #7

Master's Program in TEFL

Student teacher: Marilyn Araya

Limit Breakers English Club: An Online English Course for Mathematics Students

Assistants: Randall Montero & Pamela Fallas



Unit # 2: Delivering Arithmetic, Algebra, and Geometry Lessons

Unit Goal: At the end of the unit, students will be able to explain basic mathematical procedures related to arithmetic, algebra, and geometry by using the correct vocabulary and expressions, modal verbs, sequencing markers, and intelligible pronunciation.

General Objective: By the end of the lesson, students will be able to accurately organize the steps of mathematical procedures in the correct sequence in spoken form by using sequencing words and modal verbs.

Specific Objectives: By the end of this lesson, students will be able to...

1. accurately pronounce the vocabulary words related to equations by reading mathematical expressions aloud.
2. complete sentences with the missing information in the steps to solve an equation by listening to an explanation.
3. organize the steps of the solution of a quadratic equation in the correct sequence when interacting with each other.
4. create an explanation about equations using the vocabulary studied by observing a set of pictures and listening to each other.

5. give clear indications for steps of mathematical procedures in writing by using the modal verbs *should*, *have to*, and *need to* in the correct present or past form.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
1	<p>T welcomes the class and asks early attendees to share one thing that makes them happy.</p> <p>Warm-up: T asks students to read some numbers, fractions, powers, roots, and operations to review the vocabulary words and expressions studied in the last class. Implicitly, T demonstrates how to use modal</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>To participate in the activity:</p> <p>I can try!</p> <p>I'll do it!</p> <p>I have a question.</p> <p>I don't know how to pronounce this word.</p> <p>I think it is_____.</p> <p>I am not sure.</p>		S	15 min

	<p>verbs and sequencing words while performing the activity.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 1</p>				
<p>2</p>	<p>Pre-task 1: Ss listen to a recording of a teacher explaining equations using sequencing words and modal verbs. Ss listen to the recording two times and complete a listening comprehension exercise individually.</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>To participate in the activity:</p> <p>I can try!</p> <p>I'll do it!</p>		<p>L</p>	<p>15 min</p>

	<p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: recording, Handout 2</p>	<p>I have a question.</p> <p>I don't know how to pronounce this word.</p> <p>I think it is_____.</p> <p>Vocabulary: undo, coefficient, cancel each other out, constant, variable, operator, need to, should, inverse, simplify, numerator</p>			
<p>3</p>	<p>Task: Ss work in pairs to organize the steps to solve a quadratic equation. Once the solution is organized, Ss take turns to practice explaining each step aloud.</p> <p>Materials: Handout 3</p>	<p>To talk to classmates:</p> <p>I am not sure...</p> <p>I think the _____ (color) strip goes before/after the _____ (color) strip.</p> <p>I agree... I disagree</p> <p>I think it is...</p> <p>What do you think?</p> <p>Vocabulary:</p>		<p>S</p>	<p>20 min</p>

		undo, square root, inverse operations, squaring (raising to the power 2), have to, should.			
4	<p>Post-task: T shows a sequence of pictures showing the steps to solve two algebraic operations. Taking turns, Ss must describe the steps of a solution using the vocabulary studied, the sequencing words, and the modal verbs. Ss must be attentive to follow the steps properly.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 4</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>To participate in the activity:</p> <p>I can try!</p> <p>I'll do it!</p> <p>I need more time to think about it...</p> <p>I am not sure...</p> <p>Vocabulary:</p> <p>undo, square root, inverse operations, squaring (raising to the power 2), have to,</p>		S	10 min

		should.			
5	<p>Asynchronous Session:</p> <p>Using handout 5, Ss write sentences that describe mathematical steps and include modal verbs such as <i>should</i>, <i>have to</i>, and <i>need to</i> in present and past tense forms.</p> <p>Feedback:</p> <p>In the next class, ATs and T ask volunteers to share their sentences. T and ATs provide feedback as needed.</p> <p>Materials: Handout 5</p>	<p>Vocabulary:</p> <p>should, should have, have to, need to, had to.</p>		W	30 min

Abbreviations to be used: T = Teacher

ATs = Assistant Teachers

Ss = students

min = minutes

Lesson Plan – Week 8

University of Costa Rica

Date: 10/3/2023

Lesson Plan # 8

Master's Program in TEFL

Student teacher: Randall Montero

Limit Breakers English Club: An Online English Course for Mathematics Students

Assistants: Marilyn Araya & Pamela Fallas



Unit # 2: Delivering Arithmetic, Algebra, and Geometry Lessons

Unit Goal: At the end of the unit, students will be able to explain basic mathematical procedures related to algebra and geometry using the correct vocabulary and expressions, modal verbs, sequencing markers, and intelligible pronunciation.

General Objective: By the end of the lesson, students will be able to accurately explain two-step algebraic equations using previously learned vocabulary, appropriate sequencing words and modal verbs, and intelligible pronunciation.

Specific Objectives: By the end of this lesson, students will be able to...

1. demonstrate accurate pronunciation of algebra vocabulary by reading key terms aloud.
2. demonstrate correct understanding of a listening passage related to algebra by completing sentences with missing information.
3. construct explanations of two-step equations using the correct mathematics vocabulary and sequencing language by simulating a mathematics teacher preparing for a class about basic algebra.

4. demonstrate correct pronunciation of the vowel sound / ɪ / in algebra vocabulary by reading aloud key terms containing this sound.
5. demonstrate correct understanding of the different uses of *can* and *could* by giving examples in which these modal verbs are used.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
------------	------------	----------	------------	--------------	------

<p>1</p>	<p>Ice-breaker:</p> <p>T welcomes the class and asks Ss to share what they prefer, dogs or cats.</p> <p>Warm-up:</p> <p>Ss read aloud vocabulary items studied in previous classes from a roulette wheel to review the correct pronunciation of each of them.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials:</p> <p>Wordwall https://wordwall.net/es/resource/21630689</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>I get it now. Thank you!</p> <p>To participate in the activity:</p> <p>I can try!</p> <p>It is pronounced as...</p> <p>It is pronounced like this...</p> <p>I am not sure how to pronounce that word.</p> <p>Vocabulary:</p> <p>algebra, equation, undo, reverse, operations, coefficient, five, three, should, can, need to, both sides, unknown value</p>		<p>S</p>	<p>5 min + 10 min</p>
----------	---	---	--	----------	-------------------------------

2	<p>Pre-task 1:</p> <p>Ss listen to a listening passage about algebraic equations three times. They complete a filling in the blanks exercise (handout 1) based on the information they hear.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials:</p> <p>Handout 1</p> <p>Video: Algebra Basics= Solving 2-Step Equations - Math Antics</p>	<p>To participate in the activity:</p> <p>I can try!</p> <p>I think the answer is...</p> <p>I am not sure.</p> <p>I do not know how to pronounce this word.</p> <p>Vocabulary:</p> <p>algebra, equation, undo, reverse, operations, coefficient, five, three, should, can, need to, both sides, unknown value, algebra</p>		L	20 min
---	--	--	--	---	--------

<p>3</p>	<p>Task:</p> <p>In small groups (3-4), Ss write explanations of two-step equations using the correct mathematics vocabulary, modals, and sequencing language (handout 2). After that, they read the explanations aloud to check their pronunciation. Finally, back in the main session, Ss volunteer to read their explanations out loud.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials:</p> <p>Handout 2</p>	<p>To talk to classmates:</p> <p>What do you think?</p> <p>Do you know what _____ means?</p> <p>I don't remember how to say [Spanish word] in English.</p> <p>How do you say [Spanish word] in English?</p> <p>We can say that...</p> <p>We can add that...</p> <p>I am not sure...</p> <p>I agree I disagree</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>I need your help.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind</p>		<p>W</p> <p>S</p>	<p>40 min</p>
-----------------	--	--	--	-------------------	---------------

		<p>explaining again?</p> <p>Vocabulary:</p> <p>algebra, equation, operation, solve, addition, subtraction, multiplication, division, undo, cancel, reverse, exponent, simplify</p>			
--	--	---	--	--	--

<p>4</p>	<p>Post-task:</p> <p>In small groups (3-4), Ss take turns to read aloud the list of words in handout 3 to identify the vowel sound they have in common. Then Ss highlight where they believe the common sound occurs in each word. As a class activity, T asks Ss to volunteer to say which the common sound is and pronounce two or three example words from the exercise. Then T plays YouTube video Vowel Sound / ɪ / as in "it". After watching the video, T asks Ss to volunteer to pronounce the words in handout 3.</p> <p>ATs and T model the instructions for</p>	<p>To talk to classmates:</p> <p>How do you pronounce this word?</p> <p>Are you sure that is the correct pronunciation?</p> <p>I want to try.</p> <p>What do you think the sound in common is?</p> <p>I think it is the sound__?</p> <p>Do you think it is the sound __?</p> <p>I am not sure...</p> <p>I agree I disagree</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p>		<p>S</p>	<p>20 min</p>
----------	---	--	--	----------	---------------

	<p>students, answer questions, and provide feedback as needed.</p> <p>Materials:</p> <p>Handout 3</p> <p>YouTube video</p>	<p>Vocabulary:</p> <p>division, algebra, simplify, surface, arithmetic, addition, multiplication, reverse geometry, exponent</p>			
--	---	---	--	--	--

5	T shows an example of the Glossary Project and provides the respective due date. ATs and T answer questions as needed.	To ask the teacher: I have a question. Would you please repeat that? I did not understand. Would you mind explaining again?			5 min
---	--	---	--	--	-------

<p>6</p>	<p>Asynchronous post-task:</p> <p>Ss watch a video about the modal verbs <i>can</i> and <i>could</i>. After that, Ss write sentences or questions related to mathematics using the modal verbs. For example, when providing clear instructions for steps of mathematical procedures in writing by using the modal verbs.</p> <p>Feedback:</p> <p>In the next class, ATs and T ask volunteers to share their sentences. T and ATs provide feedback as needed.</p> <p>Materials:</p> <p>Handout 4 / YouTube video</p>	<p>Vocabulary:</p> <p>can, could</p>		<p>W</p>	<p>30 min</p>
-----------------	--	---	--	----------	---------------

Abbreviations to be used: T = Teacher ATs = Assistant Teachers Ss = students BORs = breakout rooms min =

Lesson Plan – Week 9

University of Costa Rica

Date: 10/10/2023

Lesson Plan # 9

Master's Program in TEFL

Student teacher: Pamela Fallas

Limit Breakers English Club: An Online English Course for Mathematics Students

Assistants: Marilyn Araya & Randall Montero



Unit # 2: Delivering Arithmetic, Algebra, and Geometry Lessons

Unit Goal: At the end of the unit, students will be able to explain basic mathematical procedures related to arithmetic, algebra, and geometry by using the correct vocabulary and expressions, modal verbs, sequencing markers, and intelligible pronunciation.

General Objective: By the end of the lesson, students will be able to complete a reading and listening quiz.

Specific Objectives: By the end of this lesson, students will be able to...

1. define algebra-related terms by completing a matching exercise and answering questions in an interactive online platform.
2. demonstrate understanding of basic vocabulary and expressions related to algebra by completing a reading and listening quiz based on explanations and descriptions of mathematical operations and procedures.
3. demonstrate understanding of the different uses of *can* and *could* by giving examples in which these modal verbs are used.
4. recognize vocabulary words related to geometry by making a list of words after listening to an audio.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
<p>1</p>	<p>Ice-breaker: T welcomes the class and asks Ss to share their current favorite song.</p> <p>Warm-up: T asks for volunteers to complete a matching exercise about algebra vocabulary and their corresponding definitions. AT and T model the instructions for Ss, answer questions, and provide feedback as needed.</p> <p>Materials: Activity 1</p>	<p>To participate in the activity: I can try! I think the answer is... I am not sure, but I want to try... I do not remember how to pronounce this word.</p> <p>Vocabulary: parenthesis, power, arithmetic, multiplication, fraction, equation, exponent, algebra, unknown.</p>	<p>Schema activation</p>	<p>S</p>	<p>5 + 10 min</p>

1	<p>Pre-task 1:</p> <p>Ss practice algebra-related vocabulary by answering questions using the platform mentimeter.</p> <p>AT and T model the instructions for Ss, answer questions, and provide feedback as needed.</p> <p>Materials: Mentimeter</p> <p>https://www.menti.com</p>	<p>Vocabulary: base, exponent, indices, index, integer, operator, cubed, squared, raised to the power of.</p>		S	20 min
3	<p>Task:</p> <p>T provides instructions for reading comprehension quiz #2. Ss complete reading comprehension quiz #2. After that, T provides instructions for</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>I need your help.</p> <p>Could you please repeat that?</p> <p>I did not understand. Could you explain</p>		R L	40 min

	<p>listening comprehension quiz #2. Ss complete listening comprehension quiz #2.</p> <p>Materials:</p> <p>Reading quiz #2</p> <p>Listening quiz #2</p>	<p>that again?</p> <p>I'm done.</p>			
<p>4</p>	<p>Post-task:</p> <p>T asks for volunteers to share their homework and provides feedback as needed.</p> <p>Materials:</p> <p>Handout 1</p>	<p>Vocabulary:</p> <p>can, could</p> <p>To participate in the activity:</p> <p>I can try!</p> <p>I'll do it!</p> <p>I have a question.</p> <p>I don't know how to pronounce this word.</p> <p>I think it is_____.</p> <p>I am not sure.</p>		<p>W</p>	<p>10 min</p>

<p>5</p>	<p>Asynchronous post-task: In preparation for the next class, Ss watch a video about geometry and fill out a chart by noting down words or phrases they are familiar with and those that are new to them.</p> <p>Feedback:</p> <p>In the next class, ATs and T ask volunteers to share their sentences. T and ATs provide feedback as needed.</p> <p>Materials: Handout 2</p> <p>Video:</p> <p>https://www.youtube.com/watch?v=DGKwdHMiqCg</p>	<p>Vocabulary: angles, plane, dimensional, lines, parallel, points, intersection</p>			<p>30 min</p>

Abbreviations to be used: T = Teacher

ATs = Assistant Teachers

Ss = students

min = minutes

Lesson Plan – Week 10

University of Costa Rica

Date: 10/17/2023

Lesson Plan # 10

Master's Program in TEFL

Student teacher: Marilyn Araya

Limit Breakers English Club: An Online English Course for Mathematics Students

Assistants: Pamela Fallas & Randall Montero



Unit # 2: Delivering Arithmetic, Algebra, and Geometry Lessons

Unit Goal: At the end of the unit, students will be able to explain basic mathematical procedures related to arithmetic, algebra, and geometry by using the correct vocabulary and expressions, modal verbs, sequencing markers, and intelligible pronunciation.

General Objective: By the end of the lesson, students will be able to effectively identify basic vocabulary and sequencing markers necessary for geometry lessons by completing listening and reading exercises based on authentic texts.

Specific Objectives: By the end of this lesson, students will be able to...

1. identify vocabulary related to geometry by completing a set of sentences in a multiple-choice game.
2. describe angles by using previously studied vocabulary related to geometry.
3. construct a conversation related to geometry between a mathematics teacher and a student by using the vocabulary studied.
4. revise the vocabulary related to angles and degrees by reading a passage and answering some comprehension questions.
5. recognize vocabulary related to triangles by making a list of words after watching a video and reading along the transcript.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
1	<p>Ice-breaker: Students answer the question: You can only eat one food again for the rest of your life. What is it?</p> <p>Warm-up:</p> <p>Ss practice geometry-related vocabulary by completing a set of sentences using the platform Mentimeter.</p> <p>AT and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Mentimeter</p> <p>https://www.menti.com/altj29p1p43p</p>	<p>Vocabulary: parallel, intersecting, angles, arc, perpendicular, lines, right, acute, obtuse, straight, complementary, supplementary.</p>		R	5 + 10 min

<p>2</p>	<p>Pre-task 1:</p> <p>In breakout rooms, Ss take turns to describe angles by using previously studied geometry-related vocabulary. AT and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Activity 1</p>	<p>Vocabulary:</p> <p>parallel, intersecting, angles, arc, perpendicular, lines, right, acute, obtuse, straight, complementary, supplementary.</p> <p>To participate in the activity:</p> <ul style="list-style-type: none"> •Angle ABC is a _____(acute, right, obtuse) angle. •Angle ABC is a _____(number) degree angle. 		<p>S</p>	<p>20 min</p>
-----------------	---	---	--	----------	---------------

<p>3</p>	<p>Task: In pairs, Ss write a conversation between a math teacher and a student. The conversation is about an explanation related to angles involving questions and answers by the student and the teacher. Finally, back in the main session, Ss volunteer to read their conversation out loud.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 2</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>I need your help.</p> <p>Would you please repeat that again?</p> <p>I did not understand. Would you mind explaining that again?</p> <p>I'm done.</p> <p>To participate in the activity:</p> <p>We can try!</p> <p>We'll do it!</p> <p>Vocabulary: parallel, intersecting, angles, arc, perpendicular, lines, right, acute, obtuse, straight, complementary, supplementary.</p>		<p>W</p> <p>S</p>	<p>40 min</p>
----------	--	---	--	-------------------	---------------

<p>4</p>	<p>Post-task: Individually, Ss read a passage about the types of angles and answer some questions based on the reading. Ss volunteer to share their answers.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 3</p>	<p>To participate in the activity:</p> <p>I can try!</p> <p>I'll do it!</p> <p>I have a question.</p> <p>I don't know how to pronounce this word.</p> <p>I think it is _____.</p> <p>The answer is _____.</p> <p>I am not sure.</p> <p>Vocabulary: parallel, intersecting, angles, arc, perpendicular, lines, right, acute, obtuse, straight, complementary, supplementary.</p>		<p>R</p>	<p>10 min</p>
<p>5</p>	<p>Asynchronous post-task:</p> <p>Ss list 5 vocabulary items they need to practice pronunciation from a YouTube video about the types of</p>	<p>Vocabulary: angles, equilateral, isosceles, scalene, right-angled triangles, straight sides, same length, rotational symmetry</p>			<p>30 min</p>

	<p>triangles.</p> <p>Feedback:</p> <p>In the next class, ATs and T ask volunteers to share their words. T and ATs provide feedback on pronunciation as needed.</p> <p>Materials: Handout 4</p> <p>Link to the YouTube video:</p> <p>https://www.youtube.com/watch?v=N_AxRP38GtzQ</p> <p>Link to the Oxford's Learner's Dictionary:</p> <p>https://www.oxfordlearnersdictionaries.com/us/</p>				
--	--	--	--	--	--

Abbreviations to be used: T = Teacher ATs = Assistant Teachers Ss = students min = minutes BORs =
Breakout Rooms

Lesson Plan – Week 11

University of Costa Rica

Master's Program in TEFL

Limit Breakers English Club: An Online English Course for Mathematics Student

Lesson Plan #11

Date: 10/24/2023

Student teacher: Randall Montero

Assistants: Marilyn Araya & Pamela Fallas

Unit # 3: Delivering Basic Geometry and Trigonometry Lessons

Unit Goal: At the end of the unit, students will be able to adequately deliver basic geometry and trigonometry lessons by explaining introductory level examples and problems using the corresponding vocabulary and sequencing markers.

General Objective: By the end of the lesson, students will be able to identify vocabulary necessary to deliver a basic trigonometry lesson (triangles) by completing listening and reading exercises adapted from authentic texts.

Specific Objectives: By the end of this lesson, students will be able to...

1. successfully identify words and expressions previously studied by doing an activity on a digital platform.
2. determine vocabulary related to triangles in audio passages by completing short statements.
3. construct a simple but clear explanation of the Pythagorean theorem using the appropriate vocabulary and sequencing language by simulating a mathematics teacher preparing for a class about basic trigonometry.



4. demonstrate correct pronunciation of the consonant sound /θ/ in trigonometric vocabulary by reading aloud commonly used words and phrases containing this sound.
5. recognize vocabulary related to trigonometry by completing a chart with the terms, and their corresponding abbreviations after watching a video.

Lesson Plan

Objectives	Procedures	Language	Strategies	Macro Skills	Time
<p>1</p>	<p>Ice-breaker: T welcomes the class and asks Ss to share what their favorite color is.</p> <p>Warm-up: Ss play Quizizz individually to review the vocabulary words and expressions related to angles and degrees studied in the previous class. Ss take turns to</p>	<p>To ask the teacher: I have a question. I need your help. Could you please repeat that? I did not understand. Would you mind explaining again?</p> <p>To participate in the activity: I can try! I'll do it!</p>		<p>S</p>	<p>5 min + 10 min</p>

	<p>check their answers with the rest of the class and practice the correct pronunciation of the words.</p> <p>Materials: Quizizz https://quizizz.com/admin/quiz/6536c5ee7579447f0cc66549?source=quiz_share</p>	<p>I have a question. I don't know how to pronounce this word. I think the answer is_____.</p> <p>Vocabulary: theorem, triangle, angle, length, leg, side, hypotenuse, unknown value, right angle, obtuse angle, acute angle, degrees</p>			
<p>2</p>	<p>Pre-task 1: Individually, Ss provide the missing words to complete the written captions in Handout 1 by listening to short recordings about the Pythagorean Theorem. ATs and T model the instructions for students, answer questions, and</p>	<p>To participate in the activity: I can try! I'll do it! I have a question. I don't know how to pronounce this word. I think the answer is_____.</p> <p>Vocabulary: theorem, triangle, angle, length, leg, side, hypotenuse, unknown</p>		<p>L</p>	<p>20 min</p>

	<p>provide feedback as needed.</p> <p>Materials:</p> <p>Handout 1</p>	<p>value, right angle</p>			
<p>3</p>	<p>Task:</p> <p>Ss first watch a five-minute video about the Pythagorean theorem to get familiar with explaining and using the theorem in English. Then, in small groups (2-3), Ss write an explanation of the Pythagorean theorem using the appropriate vocabulary, modals, and sequencing language (handout 2). After that, back in the main session, Ss volunteer to read their explanations out loud.</p> <p>*To test a mixed-levels strategy</p>	<p>To talk to classmates:</p> <p>What do you think?</p> <p>Do you know what _____ means?</p> <p>I don't remember how to say [<i>Spanish word</i>] in English.</p> <p>How do you say [<i>Spanish word</i>] in English?</p> <p>We can say that...</p> <p>We can add that...</p> <p>I am not sure.</p> <p>I agree. I disagree.</p> <p>To ask the teacher:</p> <p>I have a question.</p>		<p>W</p> <p>S</p>	<p>40 min</p>

	<p>suggested in Scrivener (2012), T provides one version of the handout (handout 2.a) to higher-level Ss and another version (handout 2.b) to lower-level Ss.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials:</p> <p>Handouts 2.a and 2.b</p> <p>Math Antics - The Pythagorean Theorem - Edited</p>	<p>I need your help.</p> <p>Could you please repeat that?</p> <p>I did not understand. Would you mind explaining that again?</p> <p>Vocabulary:</p> <p>theorem, triangle, angle, length, leg, side, hypotenuse, unknown value, right angle</p>			
<p>4</p>	<p>Post-task:</p> <p>First, Ss watch a YouTube video about the pronunciation of the /θ/ consonant sound. Then, in small</p>	<p>To talk to classmates:</p> <p>How do you pronounce this word?</p> <p>Are you sure that is the correct pronunciation?</p>		<p>S</p>	<p>20 min</p>

	<p>groups (3-4), Ss read aloud a list of commonly used words and phrases containing this sound to check each other's pronunciation.</p>	<p>Say the word one more time.</p> <p>Remember to put your tongue between your teeth.</p> <p>I want to try.</p> <p>I am not sure.</p> <p>To ask the teacher:</p> <p>I have a question.</p> <p>Could you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>Vocabulary:</p> <p>theorem, length, math, arithmetic, third, both, Pythagoras, Pythagorean</p>			
<p>5</p>	<p>Asynchronous post-task:</p> <p>Ss watch a video about trigonometry</p>	<p>Vocabulary:</p> <p>sine, cosine, tangent, cotangent, secant,</p>		<p>L</p>	<p>30 min</p>

	<p>terminology and complete a chart with key terminology, abbreviations, and check the link to the dictionary entry.</p> <p>Feedback:</p> <p>Next class, ATs and T ask for volunteers to share their words. T and ATs provide feedback on pronunciation as needed.</p> <p>Materials:</p> <p>Handout 4</p> <p>Video:</p> <p>https://www.youtube.com/watch?v=gHcVGS8zxmU</p> <p>Link to online dictionaries:</p> <p>https://www.oxfordlearnersdictionaries.com/us/</p>	<p>cosecant, theta, opposite, hypotenuse, adjacent</p>			
--	---	--	--	--	--

	https://dictionary.cambridge.org/us/				
--	---	--	--	--	--

Abbreviations to be used: T = Teacher ATs = Assistant Teachers Ss = students BORs = breakout rooms min = minutes

References

Scrivener, J. (2012). *Classroom management techniques: Cambridge handbook for language teachers*. Cambridge University Press.

Lesson Plan – Week 12

University of Costa Rica

Master's Program in TEFL

Limit Breakers English Club: An Online English Course for Mathematics Student

Lesson Plan #12

Date: 10/31/2023

Student teacher: Pamela Fallas

Assistants: Marilyn Araya & Randall Montero

Unit # 3: Delivering Basic Geometry and Trigonometry Lessons

Unit Goal: At the end of the unit, students will be able to adequately deliver basic geometry and trigonometry lessons by explaining introductory level examples and problems using the corresponding vocabulary and sequencing markers.

General Objective: By the end of the lesson, students will be able to successfully deliver basic trigonometry lessons by using the correct vocabulary, sequencing markers, and modal verbs.

Specific Objectives: By the end of this lesson, students will be able to...

1. demonstrate accurate pronunciation of trigonometry vocabulary by reading key terms aloud.
2. correctly use vocabulary items to complete an explanation on the basic trigonometry functions.
3. construct a simple, but clear explanation of how to find a missing side in a triangle by using the correct vocabulary, sequencing markers, and modal verbs.
4. effectively identify the stressed syllables in vocabulary related to trigonometry by dragging the words to their corresponding box.



- 5. successfully recognize the meaning of the vocabulary words by completing a filling in the blanks exercise.
- 6. effectively practice the correct pronunciation of the vocabulary words by recording themselves.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
1	<p>Ice-breaker:</p> <p>T welcomes the class and asks Ss to share what their favorite holiday is.</p> <p>Warm-up:</p> <p>Ss read aloud vocabulary items studied in the previous class from a spinning wheel to review the correct pronunciation of each of them.</p> <p>ATs and T model the instructions for Ss, answer questions, and provide feedback as needed.</p>	<p>To ask the teacher:</p> <p>How do you pronounce...?</p> <p>How do you say...?</p> <p>Could you please repeat that?</p> <p>I did not understand. Would you mind explaining again?</p> <p>I get it now. Thank you!</p> <p>To participate in the activity:</p> <p>I can try!</p> <p>I am not sure how to pronounce that word, but I am going to try...</p> <p>It is pronounced...</p>			<p>5 min</p> <p>+</p> <p>10 min</p>

	<p>Materials:</p> <p>Wordwall</p> <p>https://wordwall.net/es/resource/62980260</p>	<p>Vocabulary:</p> <p>radius, opposite, adjacent, hypotenuse, value, sine (sin), cosine (cos), tangent, (tan), secant (sec), cosecant (csc), cotangent (cot), theta θ</p>			
2	<p>Pre-task 1:</p> <p>Ss watch a video of a teacher explaining what the basic trigonometric functions are and how to use them to find unknown values (trigonometric equations). Ss watch the video twice and complete sentences individually. T pauses the video (2:34) so that Ss have a chance to complete their answers. T plays</p>	<p>To check the exercise:</p> <p>I can try!</p> <p>I can read number...</p> <p>I want to answer number...</p> <p>Vocabulary:</p> <p>radius, opposite, adjacent, hypotenuse, value, sine (sin), cosine (cos), tangent, (tan), secant (sec), cosecant (csc), cotangent (cot), theta θ</p>			20 min

	<p>that first part of the video a second time and then checks the answers. T plays the rest of the video twice and checks answers with Ss. T reads the sentences prior to playing the video for the first time.</p> <p>Materials:</p> <p>Handout 1</p> <p>YouTube video (until 8:31):</p> <p>https://www.youtube.com/watch?v=bSM7RNSbWhM</p> <p>Edited video:</p> <p>https://drive.google.com/file/d/1DGhy6x3HTMZH2tQ7MBDV8HxC7proDws/view?usp=drive_link</p>				
<p>3</p>	<p>Task:</p>	<p>To talk to your classmate:</p>		<p>L</p>	<p>40 min</p>

	<p>Task A: In pairs, Ss watch a video about finding the missing side in a triangle and complete the script. After that, Ss practice the script by taking turns. Ss. check pronunciation in the online dictionary if necessary.</p> <p>Task B: Ss watch the video about finding the missing side in a triangle to get familiar with explaining this procedure. After that, Ss complete the chart explaining the procedure using the appropriate vocabulary, modals, and sequencing language.</p> <p>After that, back in the main session, Ss volunteer to read their explanations out loud.</p>	<p>I am missing the answer for number ____.</p> <p>Number __ is _____.</p> <p>Do you know what _____ means?</p> <p>I am not sure...</p> <p>I think it is...</p> <p>I don't remember how to pronounce this word.</p> <p>Let's check this word in the dictionary.</p> <p>I agree / I disagree</p> <p>Now it is your turn.</p> <p>To talk to your classmate:</p> <p>What do you think?</p> <p>Do you know what _____ means?</p> <p>I don't remember how to say [<i>Spanish word</i>] in English.</p> <p>How do you say [<i>Spanish word</i>] in</p>		<p>S</p>	
--	--	---	--	----------	--

	<p>*To test a mixed-levels strategy suggested in Scrivener (2012), T provides Ss one version of the handout (handout 2.a) to lower level Ss and another version of the handout (handout 2.b) to higher-level Ss. ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handouts 2(a), handout 2 (b)</p> <p>Video link: https://www.youtube.com/watch?v=zbB_SddM6Gs&t=2s</p> <p>Edited video: https://drive.google.com/file/d/1EmMg</p>	<p>English?</p> <p>We can say that...</p> <p>We can add that...</p> <p>I am not sure...</p> <p>I agree / I disagree</p> <p>Vocabulary:</p> <p>Hypotenuse, opposite, adjacent, sine, cosine, tangent.</p>			
--	---	---	--	--	--

	Lb4nkuYaLU_0eC0ICo0dqT84Frch/vi ew?usp=drive_link				
4	<p>Post-task:</p> <p>In pairs, Ss pronounce the words aloud to identify the stressed syllable. Ss drag the word to the corresponding box.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Once Ss are back in the main room, T asks for volunteers to read the answers and provides feedback as needed.</p> <p>Materials: Handout 3</p>	<p>Vocabulary:</p> <p>perpendicular, geometry, trigonometry, hypotenuse, opposite, parallel, equation, angle, adjacent, direction, equal, tangent.</p> <p>Useful language:</p> <p>I think the word _____ has the stress in the (second/third) syllable from the last.</p> <p>Let's check this word in the dictionary.</p>		S	20 min
5	Asynchronous post-task: Ss review	Vocabulary:		R	30 min

	<p>the vocabulary words studied in this class by completing a filling in the blanks exercise to apply the words in context. Additionally, they review the pronunciation of the words by recording themselves pronouncing the words properly.</p> <p>Feedback:</p> <p>In the next class, ATs and T ask volunteers to share their answers. T and ATs provide feedback on pronunciation as needed.</p> <p>Materials: Handout 4</p>	<p>perpendicular, complementary, supplementary, hypotenuse, opposite, parallel, equation, angle, adjacent, direction, equal, intersection, opposite, adjacent, sine, cosine, tangent.</p>		<p>S</p>	
--	---	---	--	----------	--

--	--	--	--	--	--

Abbreviations to be used: T = Teacher ATs = Assistant Teachers Ss = students BORs = breakout rooms min = minutes

Lesson Plan – Week 13

University of Costa Rica

Master's Program in TEFL

Limit Breakers English Club: An Online English Course for Mathematics Student

Lesson Plan #13

Date: 11/07/2023

Student teacher: Marilyn Araya

Assistants: Pamela Fallas & Randall Montero

Unit # 3: Delivering Basic Geometry and Trigonometry Lessons

Unit Goal: At the end of the unit, students will be able to adequately deliver basic geometry and trigonometry lessons by explaining introductory level examples and problems using the corresponding vocabulary and sequencing markers.

General Objective: By the end of the lesson, students will be able to successfully deliver basic geometry and trigonometry lessons by using the correct vocabulary, sequencing markers, and modal verbs.

Specific Objectives: By the end of this lesson, students will be able to...

1. determine the accurate pronunciation, stress and definition of words related to trigonometry by playing a jeopardy game.
2. use vocabulary related to trigonometry by filling in the blanks with the information they hear from the audio passages.
3. demonstrate clear understanding of basic vocabulary and expressions related to trigonometry by completing a reading and listening quiz based on explanations and descriptions of mathematical operations and procedures.
4. determine the correct pronunciation of vocabulary related to algebra and geometry previously studied in class by completing a pronunciation exercise.



5. accurately explain a procedure by using the order of operations and sequencing words.

Objectives	Procedures	Language	Strategies	Macro Skills	Time
<p>1</p>	<p>Ice-breaker: T welcomes the class and asks Ss: If you could live in a different country for a year, which country would you choose?</p> <p>Warm-up: In teams, Ss play a jeopardy game where they will be answering questions about the pronunciation, syllable stress, and definition of words related to trigonometry.</p>	<p>To participate in the activity: I choose <u>(definition, pronunciation, stress)</u> for <u>(100, 200, 300, 400, 500)</u> points. I think the answer is_____ The answer is definitely _____ I'm pretty sure the answer is_____.</p> <p>Vocabulary: opposite, adjacent, hypotenuse, sine (sin), cosine (cos), tangent, (tan), secant (sec), cosecant (csc), cotangent (cot).</p>		<p>S</p>	<p>5 min + 15 min</p>

	<p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Jeopardy game</p> <p>Link: https://jeopardylabs.com/play/trigonometry-973</p>				
2	<p>Pre-task 1: Individually, students listen to audios and fill in the blanks with the information they hear.</p> <p>T asks for volunteers to go over the answers and provides feedback as needed.</p> <p>Materials: Handout 1</p>	<p>To participate in the activity:</p> <p>I can try!</p> <p>I'll do it!</p> <p>I have a question.</p> <p>I don't know how to pronounce this word.</p> <p>I think it is_____.</p>		L	20 min

<p>3</p>	<p>Task: T provides instructions for reading comprehension quiz #3.</p> <p>Students complete reading comprehension quiz #3.</p> <p>T provides instructions for listening comprehension quiz #3. Students complete listening comprehension quiz #3.</p> <p>Materials: Listening quiz #3, reading quiz #3</p>	<p>To ask the teacher:</p> <p>I have a question.</p> <p>Would you please repeat that?</p> <p>I did not understand. Would you mind explaining that again?</p>		<p>L R</p>	<p>40 min</p>
<p>4</p>	<p>Post-task:</p> <p>Working in small groups, Ss take turns to pronounce and classify the list of words in handout 2 based on sounds studied in previous classes.</p> <p>ATs and T model the instructions for</p>	<p>Vocabulary:</p> <p>three, exponent, obtuse, sum, revision, operation, arithmetic, reverse, acute, plus, conversion, fraction, addition, equation, multiplication, rule, surface, length,</p> <p>Useful language:</p>		<p>S</p>	<p>20 min</p>

	<p>students, answer questions, and provide feedback as needed.</p> <p>Materials:</p> <p>Handout 2</p>	<p>I can try!</p> <p>I can read column number...</p> <p>I don't know how to pronounce this word correctly.</p> <p>I am not sure about the pronunciation of this word.</p>			
<p>5</p>	<p>Asynchronous post-task:</p> <p>Asynchronously, Ss complete handout 3 about solving problems using order of operations and sequencing language.</p> <p>Feedback:</p> <p>In the next class, ATs and T ask volunteers to share their answers. T and ATs provide feedback on pronunciation as needed.</p>	<p>Vocabulary:</p> <p>first, next, then, after that, last, finally, before, after, add, subtract, divide, multiply, exponents, simplify</p>		<p>W</p> <p>S</p>	<p>30 min</p>

	Materials: Handout 3				
--	--------------------------------	--	--	--	--

Abbreviations to be used: T = Teacher ATs = Assistant Teachers Ss = students BORs = breakout rooms min= minutes

Lesson Plan – Week 14

University of Costa Rica

Master's Program in TEFL

Limit Breakers English Club: An Online English Course for Mathematics Professors

Lesson Plan #14

Date: 11/14/2023

Student teacher: Randall Montero

Assistants: Marilyn Araya & Pamela Fallas



Final Oral Exam: Reading mathematical operations and equations aloud

Goal: The goal of the final oral exam is for students to demonstrate their ability to deliver basic mathematical lessons related to algebra, geometry, and trigonometry by explaining introductory level problems and equations using relevant vocabulary and sequencing language correctly and pronouncing key terms intelligibly.

General Objective: By the end of the lesson, students will have demonstrated that they are able to read mathematical operations and equations clearly by using vocabulary, sequencing language, and context-specific structures.

Specific Objectives: By the end of this lesson, students will be able to...

1. determine the correct pronunciation of words related to algebra, geometry, and trigonometry by reading them aloud.
implement previously learned vocabulary words and expressions related to algebra, geometry and trigonometry correctly by describing the steps of basic mathematical operations and problems.

2. determine vocabulary, sequencing language, and context-specific structures correctly when reading aloud mathematical operations and equations clearly and intelligibly.

Lesson Plan

Objectives	Procedures	Language	Strategies	Macro Skills	Time
-------------------	-------------------	-----------------	-------------------	---------------------	-------------

<p>1</p>	<p>Ice-breaker:</p> <p>T welcomes the class and asks Ss: What is your favorite sport or physical activity?</p> <p>Warm-up: Using a slide show, Ss practice the correct pronunciation of some vocabulary items related to algebra, geometry, and trigonometry by reading them aloud.</p> <p>ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 1</p>	<p>To ask the teacher:</p> <p>I have a question. Would you please repeat that? I did not understand. Would you mind explaining that again?</p> <p>To participate in the activity:</p> <p>I can try! I'll do it! I do not remember how to pronounce this word, but I am going to try... I am not sure how to pronounce this word, but I want to try...</p> <p>Vocabulary: opposite, adjacent, hypotenuse, sine (sin), cosine (cos), tangent, (tan), secant (sec), cosecant (csc), cotangent (cot), addition,</p>		<p>S</p>	<p>5 min + 10 min</p>
----------	--	--	--	----------	-------------------------------

		subtraction, division, multiplication, angle, equation, value, triangle, first, then, next, after that, finally, add, multiply, subtract, divide			
--	--	---	--	--	--

<p>2</p>	<p>Pre-task 1: In pairs, Ss take turns to describe the steps to solve problems using the order of operations correctly, and describe some basic geometry and trigonometry problems. ATs and T model the instructions for students, answer questions, and provide feedback as needed.</p> <p>Materials: Handout 2</p>	<p>To participate in the activity:</p> <p>I can try!</p> <p>I'll do it!</p> <p>I have a question.</p> <p>I don't know how to pronounce this word.</p> <p>I don't remember how to pronounce this word.</p> <p>I think it is_____.</p> <p>Vocabulary: opposite, adjacent, hypotenuse, sine (sin), cosine (cos), tangent, (tan), secant (sec), cosecant (csc), cotangent (cot), addition, subtraction, division, multiplication, equation, value, triangle, first, then, next, after that, finally, add, multiply, subtract, divide</p>		<p>S</p>	<p>20 min</p>
----------	--	--	--	----------	---------------

NA	Mixed-levels Student Survey Ss will be given 5-10 min to complete a survey about the use of mixed-level strategies in their class.			10 min
3	Task: Final Oral Exam Individually, Ss will be provided with solution procedures that include sequences of mathematical operations required to solve different equations. They will then be asked to read each step or mathematical operation out loud (also written in mathematical symbols) using appropriate vocabulary and sequencing language, along with any other relevant structures such as modal verbs. They will not be given feedback during or after Ss read the	To ask the teacher: I have a question. I need your help. Could you please repeat that? I did not understand. Would you mind explaining again? Vocabulary: first, then, next, after that, finally	S	80 min [20 min each]

	steps out loud because this is a form of summative assessment. Materials: Final Oral Exam				
--	--	--	--	--	--

Abbreviations to be used: T = Teacher ATs = Assistant Teachers Ss = students BORs = breakout rooms min = minutes

UNIT 1: Delivering Arithmetic, Algebra, and Geometry Input

Lesson 2 - HANDOUT #1

Instructions: Read the statements below. Then listen attentively to an explanation about algebra. Complete the following statements with the missing words. You will listen to the audio twice.

USEFUL LANGUAGE	
To talk to classmates: <ul style="list-style-type: none">▪ What is the word for...?▪ What is the pronunciation of...?▪ I am not sure about this...▪ How do you say...?▪ I think it is...	To ask the teacher: <ul style="list-style-type: none">▪ I have a question.▪ Would you please repeat that?▪ I did not understand. Would you mind explaining again?

6. Algebra follows the same rules and operations as _____.
7. Algebra and arithmetic use the same four main operations called _____, _____, _____, _____.
8. In an equation, the things on both sides of the equal sign have the same _____.
9. Letters of the alphabet are used as a _____ to represent unknown numbers.
10. The mathematical statement that two things are equal is called _____.
11. Solving an equation means to figure out the _____ value.

Lesson 2 – Handout #2

Instructions	Useful Language	
<ul style="list-style-type: none"> • Drag the concepts below to match them with their corresponding definitions. 	<ul style="list-style-type: none"> • What do you think? • I'm not sure about this... • I think it is ... / I don't think it is ... • I agree... 	<ol style="list-style-type: none"> 1. A symbol (generally a letter of the alphabet) that is used to represent an unknown number
		<ol style="list-style-type: none"> 2. A combination of variables and numbers using any of the operations of addition, subtraction, multiplication, or division, as well as exponents
		<ol style="list-style-type: none"> 3. A statement that two algebraic expressions are equal
		<ol style="list-style-type: none"> 4. A way to express a repeated multiplication
		<ol style="list-style-type: none"> 5. A process in which a number or quantity is changed by adding, multiplying, subtracting, or dividing
		<ol style="list-style-type: none"> 6. The result of division
		<ol style="list-style-type: none"> 7. The result of subtraction
		<ol style="list-style-type: none"> 8. The result of addition
		<ol style="list-style-type: none"> 9. The result of multiplication

Sum

Equation

Algebraic expression

Product

Quotient

Variable

Difference

Exponents

Operation

Lesson 2 – Handout # 3

Instructions. Choose a role: Student A (math teacher) or Student B (student). Fill in the blanks to complete the conversation by using the vocabulary and definitions learned in the previous tasks.

USEFUL LANGUAGE
<ul style="list-style-type: none"> ▪ What do you think? ▪ Do you know what _____ means? ▪ What does _____ mean? ▪ I am not sure... ▪ I think it is... / I don't think it is... ▪ I agree. / I disagree.

WORD BANK
x, subtraction, algebra, addition, $3+5=x$, multiplication, placeholder, addition, division, y, $3+5=$___

ROLE-PLAY SCRIPT: ALGEBRA BASICS	
1	<p>Student A: (Informal, but respectful greeting to the class - not in the word bank)</p> <p>_____.</p> <p>In today's lesson we are going to learn about a branch of math called _____.</p> <p>Algebra is a lot like arithmetic. Why? Anyone?</p>
2	<p><u>Student B</u>: Well, it follows all the rules of arithmetic and it uses the same four main operations _____, _____, _____, _____.</p>
3	<p>Student A: Yes! So what is the difference between algebra and arithmetic?</p>
4	<p><u>Student B</u>: Algebra uses new symbols such as _____ and _____.</p>
5	<p>Student A: Exactly! It introduces the element of the unknown. In arithmetic, the only thing that was ever unknown was the answer to the problem. For example, _____.</p> <p>And we write the answer when we do the _____.</p> <p>But how do we write an equation in Algebra?</p>
6	<p><u>Student B</u>: I think that in Algebra you write it like this _____.</p>

7	Student A: Very good! The x is a _____ that represents the number or value that we don't know yet. So what we have here is a very basic algebraic equation. Who remembers the definition of an equation that we studied yesterday?
8	<u>Student B:</u> (Briefly explain what an equation is) An equation is _____.
9	Student A: Good job! You have all the good answers! So what is one of the main objectives in Algebra?
10	<u>Student B:</u> (Briefly explain one of the main objectives in Algebra) One of the main objectives is to _____.

Adapted from Math Antics

USEFUL LANGUAGE

- What do you think?
- Do you know what _____ means?
- What does _____ mean?
- I am not sure...
- I think it is... / I don't think it is...
- I agree. / I disagree.

WORD BANK

x, subtraction, algebra, addition, $3+5=x$, multiplication, placeholder, addition, division,

y, $3+5=$ _____

Lesson 2 – Handout #4

Instructions: Watch the video about the sounds “shun” [ʃən] and “zhun” [ʒən].

Video #1:

<https://www.youtube.com/watch?v=RxaLKZPPEvY>

Video #2:

<https://www.youtube.com/watch?v=wINb4HFquck&t=604s>

Video #3:

<https://www.youtube.com/watch?v=k8ImSmVOSVA>

“shun” [ʃən]	“zhun” [ʒən]
✓ Multiplication	✓ Vision
✓ Addition	✓ Division
✓ Subtraction	✓ Equation
✓ Operation	✓ Decision
✓ Pronunciation	✓ Conversion
✓ Fraction	✓ Precision
✓ Attention	✓ Conclusion
✓ Definition	✓ Revision
✓ Quotient	
✓ Ratio	

Lesson 3 – Handout #1

Instructions: Read the statements below. Then listen attentively to an explanation about the order of operations. Complete the following statements with the missing words. You will listen to the audio two times.

USEFUL LANGUAGE
To ask the teacher: <ul style="list-style-type: none">▪ What is the word for...?▪ What is the pronunciation of...?▪ I am not sure about this...▪ How do you say...?▪ Would you please repeat that?▪ I think the answer is...

1. Order of operations helps us to _____ on the order that we should do math operations and get the _____ answer.
2. The first rule is about doing the _____ and _____.
3. The second rule is about doing the _____.
4. The third rule is about doing the _____ and _____.
5. The last rule is about doing the _____ and _____.
6. The _____ are symbols that form a group of numbers and operators between them.
7. To _____ a problem means to reduce an expression in a simpler form.
8. The way of writing repeated multiplications is called _____.

Lesson 3 – Handout #2

Instructions:

1. Choose a role: Student A or Student B.
2. Organize the conversation in logical order.
3. Role-play the conversation.
4. Switch roles.

Useful Language

- What do you think?
- Do you know what _____ means?
- I am not sure...
- I think the _____ (color) strip goes before/after the _____ (color) strip.
- I agree... I disagree
- Colors: purple, gray, brown, dark green, light green, orange, blue, yellow, pink, red.

Student A: Yes, this little number on the right corner that tells how many times to multiply the base by itself.
Student B: OHH a power.

Student A: It is very simple. The order of operations is a set of rules that tell you which math operations to do first.
Student B: Which one should I do first?

Student A: I mean any operations inside parenthesis (), brackets [], or braces {}.
Student B: Now I get it. So what do I do after solving the grouping symbols?

Student A: Hmm! Let me take a look! You need to use the order of operations to solve it.
Student B: Order of operations? What is that?

Student B: Thank you so much for your help.
Student A: Anytime! See you around!
Student B: Bye!

Student A: After solving the grouping symbols, you evaluate the exponents.
Student B: Exponents?

Student A: Hello _____ (name) what are you working on?
Student B: Some math homework, but I am struggling with these expressions. I do not know how to solve them.

Student A: First, you do any operations inside the grouping symbols.
Student B: Wait! What are grouping symbols?

Student A: YESS! You can call it a power, exponent or index.
Student B: I think I remember what to do after evaluating the exponent. I should multiply and divide from left to right.

Student A: Correct! Finally, you add and subtract from left to right.

Describe the steps we need to take to solve the following problems using the order of operations. To describe each step, use one sequencing word. Sequencing language is words that allow us to arrange, and organize actions in the correct order.

Example 1: $(3^2 \times 4) + 6$

Step 1: FIRST, we do 3 to the power of 2, so we multiply 3 x 3, which is 9.

Step 2: THEN, we do the operation inside the parentheses BEFORE we do any operation outside parentheses. For this operation, we multiply 9 x 4, which equals 36.

Step 3: LAST, we add 36 + 6, which is 42.

Example 2: $10 \times (4 + 5)$

Step 1: FIRST, we add 4 + 5, which equals 9.

Step 2: THEN, we multiply 10 x 9, which is 90.

1) $30 \div 5 + (5 \times 6) \times 19 + 4$

Step 1: $(5 \times 6) = 30$ $30 \div 5 + 30 \times 19 + 4$

Step 2: $30 \div 5 = 6$ $6 + 30 \times 19 + 4$

Step 3: $30 \times 19 = 570$ $6 + 570 + 4$

Step 4: $6 + 570 = 576$ $576 + 4$

Step 5: $576 + 4 = 580$ 580

2) $35 \div 7 \times (7 - 4) + 2$

Step 1: $(7 - 4) = 3$ $35 \div 7 \times 3 + 2$

Step 2: $35 \div 7 = 5$ $5 \times 3 + 2$

Step 3: $5 \times 3 = 15$ $15 + 2$

Step 4: $15 + 2 = 17$ 17

3) $3 - 2 + 3 + 7 \times (16 \div 8)$

Step 1: $(16 \div 8) = 2$ $3 - 2 + 3 + 7 \times 2$

Step 2: $7 \times 2 = 14$ $3 - 2 + 3 + 14$

Step 3: $3 - 2 = 1$ $1 + 3 + 14$

Step 4: $1 + 3 = 4$ $4 + 14$

Step 5: $4 + 14 = 18$ 18

4) $5 \times 2 + (11 + 8) - 18 \div 9$

Step 1: $(11 + 8) = 19$ $5 \times 2 + 19 - 18 \div 9$

Step 2: $5 \times 2 = 10$ $10 + 19 - 18 \div 9$

Step 3: $18 \div 9 = 2$ $10 + 19 - 2$

Step 4: $10 + 19 = 29$ $29 - 2$

Step 5: $29 - 2 = 27$ 27

Lesson 4 – Handout #1

Instructions: Read and color-code the following information.

Yellow: the definition of geometry.

Blue: the definition of a line and line segment.

Red: the definition of a ray.

Pink: the definition of a plane.

Green: the definition of a point.

Grey: the definition of shapes.

Purple: examples of 3d shapes.

Olive green: examples of 2d shapes.

Geometry

Geometry is the study of lines, shapes, angles, and distances. Some of the most basic elements in geometry are points, lines, planes, and shapes.

We can represent a point as a dot on a piece of paper. In geometry, points describe specific locations in space. A point has no length, width, or height. The second basic element is lines. We might know what a line is. However, there is a difference between a line, a line segment, and a ray. A one-dimensional shape and collection of points extending infinitely in either direction is a line. A part of a line that has two endpoints and is the shortest distance between them is a line segment. A ray begins at one endpoint and ends at infinity. In addition, a plane is a flat surface like a window, a sheet of paper, or a tabletop. It is a two-dimensional object because there are two dimensions that you can move in. You can go up and down, or you can go left and right.

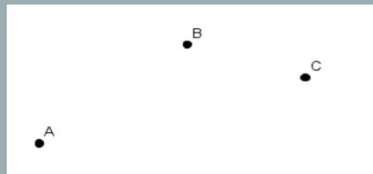
Another important element of geometry is shapes. In mathematics, shapes define the outline or the boundary of an object. Based on the dimensions of the shapes, each shape can be classified as two-dimensional shapes (2D shapes) and three-dimensional shapes (3D shapes). The names of basic 2d shapes are circle, triangle, square, rectangle, and so on. The basic 3d shapes are sphere, cube, cone, cylinder, etc.

Lesson 4 – Handout #2

GEOMETRY

Vocabulary and definitions

Instructions: Fill in the blanks with the information you hear.



A _____ in _____ is represented by a _____, which denotes its _____ in space.

To participate in the activity:

I can try!
I'll do it!
I have a question.
I don't know how to pronounce this word.
I think it is _____.



Instructions: Fill in the blanks with the information you hear.



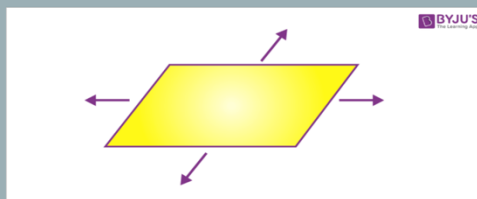
A one-dimensional _____ and collection of _____ extending infinitely in either direction is a _____.

To participate in the activity:

I can try!
I'll do it!
I have a question.
I don't know how to pronounce this word.
I think it is _____.



Instructions: Fill in the blanks with the information you hear.



A _____ is a _____ of two dimensions. A plane has _____ thickness, zero curvature, infinite _____, and infinite _____.

To participate in the activity:

I can try!
I'll do it!
I have a question.
I don't know how to pronounce this word.
I think it is _____.



Lesson 4 – Handout #3

Instructions. Choose a role: Student A (math teacher), Student B (student), Student C (student). Fill in the blanks to complete the conversation by using the vocabulary and definitions learned in the previous tasks.

Useful Language	
To talk to classmates	To ask the teacher
What do you think?	I have a question.
Do you know what _____ means?	Would you please repeat that?
We can say that...	I did not understand. Would you mind explaining again?
We can add that...	
I am not sure...	
I agree... I disagree	

WORD BANK
distance, describe, triangle, properties, relationships, lines, surfaces, solids, letters, name

ROLE-PLAY SCRIPT: GEOMETRY	
1	Student A: (Greeting to class-not in the word bank) _____. In today's lesson we are going to learn about geometry.
2	Student B: Teacher, what is geometry?
3	Student A: Good question! Geometry a branch of mathematics that deals with the measurement, _____, and _____ of points, _____, angles, _____, and solids.
	Student C: Do you mean shapes like a square or a _____?
4	Student A: Exactly! Shapes like a square or a triangle.

5	Student B: Got it! What about the lines and points that you mentioned before? What does that mean in geometry?
6	Student A: Oh right! Let me tell you about the points first. Points are really important in geometry. They help us to _____ specific locations in space, like the end of a line or the corner of a square. One more thing, we need to _____ them in order to know which one we are talking about.
8	Student C: How do we name them?
9	Student A: Easy! We should name them by using _____ such as A, B, C, or D. Now, let me tell you about the lines.
10	Student B: Teacher, may I try to explain what lines in geometry are?
11	Students A: Of course! Go ahead.
12	Student B: I think that a line is a figure formed when two points are connected with minimum _____ between them.
13	Student C: Is that correct?
14	Student A: Very good answer, thank you. We will continue learning about geometry in our next class. It's time to go home.



Lesson 4 – Homework

Homework

Practice the pronunciation of the following words. Record yourself and send it to your instructor.

Do not forget to check the correct pronunciation of the words

at: <https://dictionary.cambridge.org/es/translate/>

1. Geometry
2. Shape
3. Line
4. Point
5. Plane
6. Dot
7. Location
8. Endpoints
9. Segment
10. Ray
11. Infinitive
12. Surface

Lesson 5 – Handout #1

Task: In your group, answer the following questions. You can use pen and paper to write down your answers.

1. What is the difference between the formula in English and the formula in Spanish to find the area of a triangle?
2. Explain the procedure to calculate the area of an acute triangle that has a base of 5 meters and a height of 8 meters. What is the area of this acute triangle?
3. Explain the procedure to calculate the area of an obtuse triangle that has a base of 4 inches and a height of 7 inches. What is the area of this obtuse triangle?
4. Explain the procedure to calculate the area of another quadrilateral, the trapezoid, with a first base of 10 meters (top), a second base of 20 meters (bottom) and a height of 8 m. What is the area of this trapezoid?

Useful Language

- I want to answer question one / two / three / four.
- I can answer question one / two / three / four.
- I don't know how to say [Spanish word] in English.
- I don't remember how to say [Spanish word] in English.
- I forgot how to say [Spanish word] in English.
- How do you say [Spanish word] in English?
- I think the answer is _____.
- I think the word in English is _____.
- I think in English you say...
- In Spanish, we say..., but in English, they say...

Lesson 5 – Handout #2

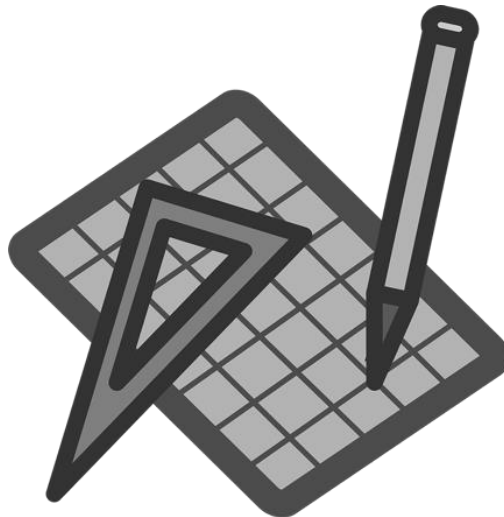
Task: In your group, read and write down the equation below correctly using what you have learned already. Do not solve the equation. 5 min

This is the equation:

$$(6-5) \times 4 + 4.7 + 2^3 - \frac{7}{10} \neq \sqrt{257}$$

Useful Language

- The first part of the equation is...
- The next part of the equation is...
- The last part of the equation is...
- I don't know how to say [Spanish word] in English.
- I don't remember how to say [Spanish word] in English.
- I forgot how to say [Spanish word] in English.
- How do you say [Spanish word] in English?
- I think the answer is _____.
- I think the word in English is _____.
- I think in English you say...
- In Spanish, we say..., but in English, they say...



A power consists of two parts: the base and the exponent. There are several ways of reading powers. We have two special cases. These are the powers where the exponent (the number written as an upper index) equals 2 or 3. With other exponents there are also basically two ways of reading them.

3^2	three squared
	the square of three

2^3	two cubed
	the cube of two

5^4	five to the power of four
	five (raised) to the fourth power

Roots

With roots we also have two special cases, very much like with powers.

$\sqrt{4}$	the square root of four
------------	-------------------------

$\sqrt[3]{8}$	the cube root of eight
---------------	------------------------

$\sqrt[4]{16}$	the fourth root of sixteen
----------------	----------------------------

Comparisons

Numbers are often compared.

$a = b$ a is equal to b

$a > b$ a is greater than b

$a < b$ a is less than b

$a \geq b$ a is greater than or equal to b

$a \leq b$ a is less than or equal to b

Operations

And here are the basic operations: addition, subtraction, multiplication and

division:

$a + b$ a plus b

$a - b$ a minus b

$a \times b$ a times b
a multiplied by b

$a \div b$ a divided by b

Adapted from ***How to Read Math – the Basics*** -
<https://prosperoenglish.com/posts/vocabulary/read-math/>

Lesson 5 – Handout #4

- **Task: Choose a role: Student A (math teacher), Student B (student), Student C (student).** Complete the conversation by using the vocabulary and definitions learned in the previous tasks. Then practice the dialogue with your classmates.

Useful Language	
To talk to classmates	To ask the teacher
What do you think?	I need your help.
Do you know what _____ means?	I have a question.
I don't remember how to say [Spanish word] in English.	Would you please repeat that?
How do you say [Spanish word] in English?	I did not understand. Would you mind explaining again?
We can say that...	
We can add that...	
I am not sure...	
I agree... I disagree	

ROLE-PLAY SCRIPT	
1	Student A: Hello. Let's first review how to calculate the area of squares, rectangles, and triangles. How do we find the area of squares and rectangles?
2	Student B: To find the area of a square or a rectangle, we need to... [<i>says the formula</i>].
3	Student A: That's correct! And how do we calculate the area of triangles?
4	Student C: To find the area of a triangle, we need to... [<i>says the BUT makes one mistake in the formula</i>].
5	Student A: Close! To find the area of a triangle, we need to... [<i>says the correct formula</i>].
6	Student C: Oh, that's right!

6	Student A: Now, let's talk about equations. How do we read the following equation? <i>[equation consists of three mathematical operations and includes one negative number and one fraction]</i>
8	Student C: I can do it! <i>[reads aloud the equation]</i> .
9	Student A: Good! And what about this other equation? <i>[equation consists of four mathematical operations and includes one square root and one power]</i>
10	Student B: I can try!
11	Students A: Of course! Go ahead.
12	Student B: <i>[reads aloud the equation]</i>
13	Student C: Is that correct?
14	Student A: It sure is!
15	Student C: How do we read powers? I don't remember.
16	Student A: Remember that a power consists of two parts: the _____ and the _____. Take for example 5^4 . There are two ways of reading this power. One way is to say _____ and the other way is to say _____.
17	Student C: I remember now. Thank you, teacher!

Lesson 5 – Handout #5

- All the words in the box include the letter u. How is it pronounced? Write the words in the table.

	1. / ʌ /	2. / u /
1. include	<i>number</i>	<i>include</i>
2. number		
3. multiplication		
4. sum		
5. root		
6. plus		
7. rule		
8. acute		
9. minus		
10. obtuse		
11. multiply		
12. numerator		
13. subtraction		

Useful Language

- What do you think?
- Do you think it is column one or column two?
- I think it is column one / two.
- Are you sure it is column one / two?
- Do you know what _____ means?
- I don't remember how to pronounce [word] in English.
- How do you say [Spanish word] in English?
- I am not sure...
- I agree... I disagree...

Lesson 5 – Handout #5

1. Watch the video below to learn about the English sound / ʌ /.

<https://www.youtube.com/watch?v=X1utTZqC3AI>

2. Watch the video below to learn about the English sound / u /.

<https://www.youtube.com/watch?v=IkM6CKBM2ns>

3. Draw arrows to connect the sentences. Make 5 two-line conversations.

1. Where should I put your luggage?
2. But I bought a new tube on Tuesday.
3. You'll be too hot in the sun.
4. My brother. Would you like me to introduce you?
5. Thanks. It's from a really good cookbook.
6. There isn't much toothpaste left.
7. In the trunk. I just have one suitcase.
8. I think I'll put on my wool suit.
9. Who's that in the blue uniform?
10. That onion soup was wonderful.



4. Underline all the words where the letter < u > is pronounced as / ʌ /. How many can you find?

5. Circle all the words where the letter < u > is pronounced as / u /. How many can you find?

Adapted from Pronunciation Plus by Martin Hewings and Sharon Goldstein

Lesson 6 – Handout #1

Asynchronous Session

Part I

Instructions: Watch the following videos. After watching the videos, practice the new vocabulary by completing the following word search.

Video 1:

<https://www.youtube.com/watch?v=NybHckSEQBI&list=PL4mRaHZim4UQPTOTHTe6uKnRoGSo5GuP4&index=1>

Video 2:

https://www.youtube.com/watch?v=Qyd_v3DGzTM&list=PL4mRaHZim4UQPTOTHTe6uKnRoGSo5GuP4&index=3

Word search: <https://www.educima.com/wordsearches/vocabulary-87141f8091989bb54e50a7168c7628b9>

Part II

Instructions: Complete the following chart with the corresponding definition and pronunciation reference from an online dictionary. The five empty spaces in the chart is for you to include any vocabulary that is new for you from the videos.

Online dictionary:

<https://dictionary.cambridge.org/us/>

Vocabulary Word	Definition	Pronunciation
Cancel something out		
Coefficient		
Equation		
Expression		
Inverse		
Isolate		
Variable		
Operator		
Reverse		
Undo		
Variable		

Lesson 6 – Handout #5

All the words in the box include the letter u. How is it pronounced? Write the words in the table.

1. include
2. number
3. multiplication
4. sum
5. root
6. plus
7. rule
8. acute
9. minus
10. obtuse
11. multiply
12. numerator
13. subtraction

1. / ʌ /	2. / u /
<i>number</i>	<i>include</i>

Useful Language
<p>To participate in the activity:</p> <ul style="list-style-type: none"> • I can try! • I'll do it! • I have a question. • I don't know how to pronounce this word. • I think it is_____. • I am not sure.

Lesson 6 – Listening Quiz

Name:

Total points: 7 pts.

Points obtained:

Listen to a mathematics professor and a student talk about *Order of Operations Rules* and answer the following questions.

1. How many rules does the student remember? Highlight the correct answer. (1 pt.)
 - a. two
 - b. three
 - c. four
 - d. five
2. Which rule specifically does the student forget? Highlight the correct answer. (1 pt.)
 - a. the rule about brackets
 - b. the rule about exponents
 - c. the rule about addition and subtraction
 - d. the left-to-right rule
3. What does the student forget about the first rule? Highlight the correct answer. (1 pt.)
 - a. exponents
 - b. roots
 - c. brackets
 - d. parentheses
4. What does the student suggest to do when we have a problem that has exponents inside parentheses? Highlight the correct answer. (1 pt.)
 - a. Simplify the exponents first.
 - b. Ignore the exponents because they are in parentheses.
 - c. Find a way to get the exponents outside the parentheses.
 - d. Solve everything else in the parentheses before simplifying the exponents.
5. Outside parentheses, which operations does the student say we should do first? Highlight the correct answer. (1 pt.)
 - a. multiplication and division
 - b. addition and subtraction
 - c. simplification of exponents
 - d. addition and multiplication

6. What does the student suggest to do when we have a problem that has multiplication and division? Highlight the correct answer. (1 pt.)
- a. Do the multiplication first and then the division
 - b. Do the division first and then the multiplication
 - c. Do the operations from left to right
 - d. Do the operations from right to left
7. According to the instructor, which math operation takes next priority after doing operations in parentheses and brackets? Highlight the correct answer. (1 pt.)
- a. operations on the left
 - b. multiplying and dividing
 - c. adding and subtracting
 - d. simplifying exponents

Lesson 6 – Reading Quiz

Name:

Total points: *10 pts.*

Points obtained:

Read the text below from an introductory class to geometry and answer the questions below.

Today we are going to start a new subject where we are going to learn the basics of a branch of mathematics called geometry. Geometry is the study of points, lines, shapes, angles, distances, and surfaces. In this class, we are going to focus on three of the most basic elements of geometry: points, lines, and planes. We are going to start with points because they are the simplest thing you can imagine in geometry. But, what's the point? Points are dots in space. They help us describe specific locations in space, like the end of a line, the corner of a square, or the center of a circle. A line is the next most basic element in geometry. You can form a line between any two points, and we name the line by the points that it goes through. For example, we can name a line A and B because its end-points (where the line starts and stops) are points A and B. A plane is a flat surface, like a window, a sheet of paper, or a door. A plane or flat surface is what we call a two-dimensional object because there are two dimensions that you can move in: you can go up and down, or you can go left and right. A triangle is an example of a two-dimensional object. But there are also three-dimensional objects. In a three-dimensional object, you can also move left to right and go up and down, but you can also go in and out. An example of a three-dimensional object is the cube. In geometry, we refer to three-dimensional objects as solids.

1. What is geometry? (*1 pt.*)
2. What are the three most basic elements of geometry? (*3 pts.*)
3. What is a point? (*1 pt.*)
4. What is a plane? (*1 pt.*)
5. What are solids? (*1 pt.*)
6. What is the difference between a plane or flat surface and a solid? (*2 pt.*)
7. What is one example of a solid?

Lesson 7 – Asynchronous Session

Instructions: Write steps by using modal verbs.

1. SHOULD (EXPRESSES ADVICE)

Check the pronunciation of *should*:

<https://dictionary.cambridge.org/us/dictionary/english/should>

- Present Tense: [Subject] + should + [infinitive verb]

Example 1: $2x + 2 = 8$

Step 1: You should undo the addition by subtracting 2 from both sides of the operation.

$$\begin{array}{r} 2x + 2 = 8 \\ - 2 \quad - 2 \\ \hline 2x = 6 \end{array}$$

Your turn: Write the next step and color code [Subject] + should + [infinitive verb]

Step 2: _____.

$$\frac{2x}{2} = \frac{6}{2}$$

- Past Tense: [Subject] + should + have + [past participle verb]

Example: You should have undone the addition.

Your turn: Your students got the wrong answer because he did not follow the order of operations in reverse. What would you say to the student? Write your piece of advice by following this structure:

[Subject] + should + have + [past participle verb]

_____.

2. HAVE TO / NEED TO (expresses necessity and obligation)

Check the pronunciation of *have to*: <https://dictionary.cambridge.org/us/dictionary/english/have-to>

Check the pronunciation of *need to*:

<https://dictionary.cambridge.org/us/dictionary/english/need?q=need+to>

- Present tense: [Subject] + have/has to/ need to + [infinitive verb]

Example: We need to get rid of the square root sign.

We have to get rid of the square root sign.

Your turn: Write the first step to solve this equation and color code [Subject] + have to/ need to + [infinitive verb]

$$X + 7 = 15$$

Step 1:

$$X + 7 = 15$$

$$-7 \quad -7$$

- Past Tense: [Subject] + had to/ needed to + [infinitive verb]
Example: We had to get rid of the square root sign.

Your turn: Write the same step but in past tense and color-code [Subject] + had to + [infinitive verb]

$$X + 7 = 15$$

Step 1:

$$X + 7 = 15$$

$$-7 \quad -7$$

Lesson 7 – Handout #1

LET'S READ MATH

INSTRUCTIONS:

Read the following numbers, fractions, powers, roots, and operations aloud.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = mx + b$

$V = \frac{4}{3}\pi r^3$

LET'S READ

- 3

- I can try!
- I have a question.
- I think it is_____.
- I don't know how to pronounce this.
- I am not sure.

$\sqrt[3]{9}$

- I can try!
- I have a question.
- I think it is_____.
- I don't know how to pronounce this.
- I am not sure.

$42 - 6 \times 2 \div 4 \times 3 + 5$

- I can try!
- I have a question.
- I think it is_____.
- I don't know how to pronounce this.
- I am not sure.

Lesson 7 – Handout #2

Handout 2

Instructions: Listen to the audio twice and complete with the information you hear.

Word bank: undo, coefficient, cancel each other out, constant, variable, operator, need to, should, inverse, simplify, numerator

Parts of the Equation

$$\boxed{}$$

$$2x - 5 = 13$$

To participate in the activity:

I can try!
I'll do it!
I have a question.
I don't know how to pronounce this word.
I think it is_____.

Step 1: First _____ operations to _____
Therefore, to _____ the subtraction, you should add 5 to both sides.

$$\begin{array}{r} 2x - 5 = 13 \\ +5 \quad +5 \end{array}$$

Step 2: After that, you _____ the expression. On the left-hand side, the fives _____ . On the right-hand side, you add $13 + 5$, which equals 18.

$$2x = 18$$

Step 3: Now to undo the multiplication, you _____ divide both sides by 2.

$$\frac{2x}{2} = \frac{18}{2}$$

Step 4: Finally, simplify the expression. On the left-hand side, cancel the terms in both the _____ and the denominator. On the right-hand side, you _____ divide 18 by 2 which simplifies to 9.

$$X=9$$

Lesson 7 – Handout #3

To talk to classmates:

- I am not sure...
- I agree/disagree.
- I think the color strip goes (before/after) the (color) strip.

Handout 3

Instructions: Organize the steps to solve the following quadratic equation $7x^2 - 11 = 0$. After that, practice with your partner explaining the steps aloud by taking turns.

Steps

The **next** step is to simplify. The plus 11 and the minus 11 on the left side cancel each other out.

$$7x^2 = 11$$

Then, since the variable x^2 is being multiplied by 7, we **have to** undo that by dividing by 7 on both sides.

$$\frac{7x^2}{7} = \frac{11}{7}$$

The 7's on the left side cancel each other out. On the right side, 11 does not divide into 7, so you **can** leave it as a fraction.

$$x^2 = \frac{11}{7}$$

Finally, you simplify leaving x all by itself. On the other side, you **should** rationalize the denominator. So the answer to this problem is $X = \sqrt{\frac{77}{7}}$

$$X = \sqrt{\frac{11}{7}}$$

$$X = \sqrt{\frac{11}{7}} \cdot \sqrt{\frac{7}{7}}$$

$$X = \sqrt{\frac{77}{7}}$$

First, we **should** know that we **have to** use inverse operations to solve this equation. We **do not have to** use factoring because there is only one variable. **The first step** is to undo what is happening to the variable. In this case, the variable is being subtracted by 11. Since the inverse operation of subtraction is addition, you **have to** add 11 to both sides.

$$7x^2 - 11 = 0$$

$$+11 \quad +11$$

After that, we need to undo the variable being squared. You can do that by using the inverse operation and taking the square root on both sides.

$$\sqrt{x^2} = \sqrt{\frac{11}{7}}$$

Lesson 7 – Handout #4



Instructions. Taking turns, read the following equations using the vocabulary studied today. You must be attentive in order to continue with the reading according to the rules of the inverse order of operations.

1

- I can try!
- I think the next step is...
- I am not sure.
- Could you please repeat that?

$$x^2 = 64$$

- I can try!
- I think the next step is...
- I am not sure.
- Could you please repeat that?

$$\sqrt{x^2} = \sqrt{64}$$

- I can try!
- I think the next step is...
- I am not sure.
- Could you please repeat that?

$$x = \sqrt{64}$$

- I can try!
- I think the next step is...
- I am not sure.
- Could you please repeat that?

$$X = 8, -8$$

Lesson 8 – Handout #1

Handout 1

Instructions:

1. Listen to an audio about algebra.
 2. Complete the following sentences with the missing words.
 3. Share your answers with your classmates.
 4. Be prepared to practice the correct pronunciation of the words.
-

1. Equations that have two arithmetic operations require _____ different steps to _____ them.
2. When there is more than one operation, we have to decide what _____ to undo the operations.
3. To _____ a two-step equation, we have to use the rules of order of operations in _____.
4. You have to pay attention to how the numbers are _____ in an equation.

To participate in the activity:

- I can try!
- I listened to the word / words...
- I don't know how to pronounce this word...
- I think I heard...
- I am not sure...

Lesson 8 – Handout #2

Handout 2

Task: Imagine that you are preparing to teach a mathematics class about basic algebra. Since this is going to be the first time you teach mathematics in English, you asked a colleague to help you write clear explanations of how to solve 2 two-step equations. Use the useful language below and any relevant concepts (vocabulary) in English learned in previous tasks to help you construct your explanations.

1. $2x + 2 = 8$

2. $\frac{x}{2} - 1 = 4$

Useful language to construct your explanations	Useful language to talk to your classmate
<ul style="list-style-type: none"> - In math... In algebra... In this equation... - We [<i>should</i> <i>can</i> <i>need to</i>] solve the unknown value 'x' by [<i>adding</i> <i>subtracting</i> <i>multiplying</i>] ... FIRST, and THEN we [<i>should</i> <i>can</i> <i>need to</i>] ... - To undo those two operations, we [<i>should</i> / <i>can</i> / <i>need to</i>] ... - THEN do the [<i>exponent</i> <i>multiplication</i> <i>division</i> <i>addition</i> <i>subtraction</i>] ... - NEXT, we [<i>should</i> <i>can</i> <i>need to</i>] ... - When solving equations, the best strategy is to... - We need to follow the Order of Operations in reverse order... - We should UNDO addition BEFORE we UNDO multiplication - FIRST, we UNDO the addition... THEN we DO the... - On the FIRST side, we [<i>should</i> <i>can</i> <i>need to</i>] ... - And on the OTHER side, we [<i>should</i> <i>can</i> <i>need to</i>] ... - To UNDO this operation, we [<i>should</i> <i>can</i> <i>need to</i>] [<i>add</i> <i>subtract</i>] to both sides of the equation. - Now we know that $x = 3$ - So our answer is $x = 10$ 	<p>What do you think?</p> <p>Do you know what _____ means?</p> <p>I don't remember how to say [<i>Spanish word</i>] in English.</p> <p>How do you say [<i>Spanish word</i>] in English?</p> <p>We can say that...</p> <p>We can add that...</p> <p>I am not sure...</p> <p>I agree / I disagree</p>

Lesson 8 – Handout #3

Handout 3

Task: With your classmates, read aloud the following words. Pay attention to your pronunciation. First, find the sound that these words have in common (which sound –not letter– do these words have in common?) Then **highlight** in each word the letter that represents the common sound. You can check the Oxford Learners Dictionary online for help:

<https://www.oxfordlearnersdictionaries.com/us/>.

arithmetic
addition
multiplication
division
simplify
reverse
exponent
algebra

To participate in the activity:
<ul style="list-style-type: none">• How do you pronounce this word?• Are you sure that is the correct pronunciation?• I want to try.• What do you think the sound in common is?• I think it is the sound ___?• Do you think it is the sound ___?• I am not sure...• I agree / I disagree

Lesson 8 – Handout #4

Asynchronous Session

Instructions: Watch the following video about the difference between the modals **can** and **could**. After that, write sentences or questions related to mathematics using these modal verbs.

Video 1: <https://www.youtube.com/watch?v=Cfhi9s7Uouo>

1. Can

- a) Ability

Example: I **can** solve quadratic equations.

Your example:

- b) Offer

Example: I **can** show you how to simplify this expression.

Your example:

- c) Possibilities

Example: Math **can** be very easy for some people.

Your example:

- d) Permission

Example: **Can** I use the calculator?

Your example:

2. Could

- a) Ability

Example: I **could** solve trigonometric equations when I was in high school.

Your example:

- b) Suggestions

Example: She **could** help you with the homework if you want.

Your example:

- c) Possibilities

Example: You **could** have solved it by using the order of operations.

Your example:

- d) Request

Example: **Could** you tell me the formula to find the area of a circle?

Your example:

- e) Permission

Example: **Could** I borrow your book?
Your example:

Good Job!!



Useful Language

I can try!
I think the answer is...
I am not sure, but I want to try...
I do not remember how to pronounce this word...

1	<u>algebra</u> is branch of mathematics in which letters can represent numbers in equations.	parentheses
2	A <u>power</u> consists of the base and the exponent.	power
3	'X' is the letter typically used to represent an <u>unknown</u> number.	arithmetic
4	An <u>equation</u> is a mathematical statement that shows that two mathematical expressions are equal.	multiplication
5	Addition, subtraction, multiplication and division are the basic <u>arithmetic</u> operations.	fraction
6	<u>multiplication</u> is the 'default' operation in algebra.	equation
7	<u>parentheses</u> are used to show groupings in math.	exponent
8	A <u>fraction</u> consists of a numerator and a denominator.	algebra
9	The <u>exponent</u> of a number says how many times to use the number in a multiplication.	unknown

Lesson 9 – Handout #1

Asynchronous Session

Instructions: Watch the following video about the difference between the modals **can** and **could**. After that, write sentences or questions related to mathematics using the modals verbs.

Video 1: <https://www.youtube.com/watch?v=Cfhi9s7Uouo>

1. Can

a) Ability

Example: I **can** solve quadratic equations.

Your example:

b) Offer

Example: I **can** show you how to simplify this expression.

Your example:

c) Possibilities

Example: Math **can** be very easy for some people.

Your example:

d) Permission

Example: **Can** I use the calculator?

Your example:

2. Could

a) Ability

Example: I **could** solve trigonometric equations when I was in high school.

Your example:

b) Suggestions

Example: She **could** help you with the homework if you want.

Your example:

c) Possibilities

Example: You **could** have solved it by using the order of operations.

Your example:

d) Request

Example: **Could** you tell me the formula to find the area of a circle?

Your example:

e) Permission

Example: **Could** I borrow your book?
Your example:

Good Job!!

Lesson 9 – Handout #2

Asynchronous Session

Instructions:

1. Watch the following video about geometry:
<https://www.youtube.com/watch?v=DGKwdHMiqCg>
2. Watch the video as many times as necessary and take notes to complete the chart below.
3. Practice the correct pronunciation of the words and be ready to share them with your classmates next class.

Words or phrases that I already know	New words or phrases

Lesson 9 – Listening Quiz #2

Listening Quiz Two

Total points: 12 pts / 5% Points obtained: ___ pts. / ___ %

Name: _____

Time allotted: 20 min

General instructions: Listen carefully twice and choose the best answer. Once finished, submit your quiz to Google Classroom. You have 20 minutes to complete and submit this quiz.

Instructions: Listen to a mathematics professor talk about **equations**. Fill in the blanks with the missing words from the box below.

placeholder, multiplication, values, symbol, game, arithmetic, division, times, unknown, parentheses, equation, default

1. Algebra is a lot like _____, but algebra introduces the element of the _____.
2. In algebra, when we don't know what a number is yet, we use a _____ in its place.
3. The 'x' is a _____ that stands for the number that we don't know yet.
4. An _____ is just a mathematical statement that two things are equal.
5. In Algebra, we solve equations to figure out the unknown _____.
6. Solving equations is a lot like a _____, and your job is to simplify them and rearrange them until they are nice, simple equations.
7. In Algebra, _____ is usually written in fraction form.
8. In Algebra, the _____ sign is usually not shown, because it's the _____ operation.
9. Whenever you need to show multiplication between two known numbers, you still have to use the _____ symbol, unless you use _____ instead.

Lesson 9 – Reading Quiz #2

Reading Quiz 2

Name: _____

Time allotted:

20 min

Total point: 10pts (5%)

Points Obtained: _____

General instructions: Read carefully and choose the best answer. Once finished, submit your quiz to Google Classroom. You have 20 minutes to complete and submit this quiz.

Part I

Instructions: Read the next passage silently to yourself. When you come to a box, read all the words in the box and highlight the word that makes the most sense to you in the context of the passage. (5pts)

An algebra
algebraic
algebraics **All about Equations** equation can be defined as a mathematical statement in which two e set equal to each other. It usually consists of a variable constants.
fractions.
consistent. coefficients and

In simple words, equations mean equality i.e. the equal sign. That's what equations are all about- "equating one quantity with another". Equations are like a balance scale. If you've seen a balance scale, you know that an difference
unequal
equal amount of weight has to be placed on either side

to be considered "balanced". If we add some weight to just one side, the scale will tip on one side and the two sides are no longer in balance. Equations follow the same logic.

Whatever is on one side of the equation value
numerator
multiply is exactly the same on the other side else it becomes an

For example, $2x+3 = 7$ is an equation, where $2x+3$ and 7 are equated by equal to "=" sign. $2x+3$ is at the Left-hand side of the equation and 7 is at the right-hand side. In this example, $2x$, 3 and 7 are terms, x is the variable, operation.
operator.
symbolic. the constants, and '+' is the

Part II

Instructions: Read the following information about exponents and then drag and drop the property of exponents with the corresponding examples. (5pts)

Exponents

A number in exponential form has a base and an exponent. In the expression a^b , a is the base and b is the exponent. This is also referred to as “ a raised to the power b .”

When the exponent is a positive integer, it indicates how many times the base is multiplied by itself. For example, $3^4 = 3 \times 3 \times 3 \times 3 = 81$. When the exponent is a negative integer, the expression is equal to the reciprocal of the expression with a positive exponent. For example, $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$.

Properties of exponents are shown in the table below

Property	Example
Multiplying like bases: To multiply two numbers with the same base, add the exponents.	
Dividing like bases: To divide two numbers with the same base, subtract the exponents.	
Power raised to a power: To raise a number in exponential form to a power, multiply the exponents.	
Product raised to a power: When a product of numbers is raised to a power, raise each part of the product to the power.	
Zero exponent: The value of any expression with a zero exponent is 1.	

Examples:

$$(x^a)^b = x^{ab}$$

$$(2^7)^3 = 2^{7 \times 3} = 2^{21}$$

$$x^a \div x^b = x^{a-b}$$

$$3^4 \div 3^9 = 3^{4-9} = 3^{-5} = \frac{1}{3^5}$$

$$(xy)^a = x^a \times y^a$$

$$(2 \times 3^4)^3 = 2^3 \times 3^{4 \times 3} \\ = 2^3 \times 3^{12}$$

$$x^a \times x^b = x^{a+b}$$

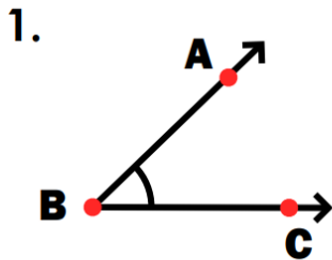
$$4^5 \times 4^2 = 4^{5+2} = 4^7$$

$$x^0 = 1$$

$$6^0 = 1$$

Describing Angles

Instructions: Describe the angle using mathematical language.



Useful Language

- Angle ABC is a _____ (acute, right, obtuse) angle.
- Angle ABC is a _____ (number) degree angle.

A-B-C-D-E-F-G-H-I

Ei bi si di I ef yi eich ai

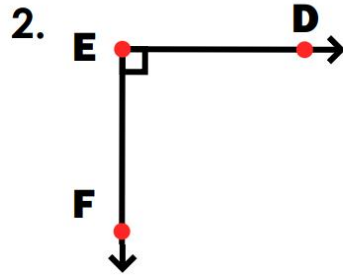
J-K-L-M-N-O-P-Q-R-

Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z

es ti iu vi dabliu ex uay zi

Instructions: Describe the angle using mathematical language.



Useful Language

- Angle DEF is a _____ (acute, right, obtuse) angle.
- Angle DEF is a _____ (number) degree angle.

A-B-C-D-E-F-G-H-I

Ei bi si di I ef yi eich ai

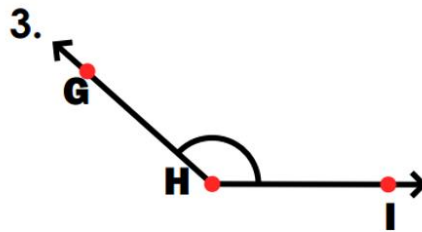
J-K-L-M-N-O-P-Q-R-

Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z

es ti iu vi dabliu ex uay zi

Instructions: Describe the angle using mathematical language.



Useful Language

- Angle GHI is a _____ (acute, right, obtuse) angle.
- Angle GHI is a _____ (number) degree angle.

A-B-C-D-E-F-G-H-I

Ei bi si di I ef yi eich ai

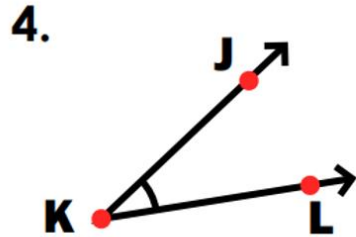
J-K-L-M-N-O-P-Q-R-

Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z

es ti iu vi dabliu ex uay zi

Instructions: Describe the angle using mathematical language.



Useful Language

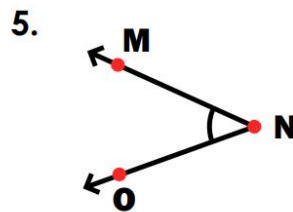
- Angle JKL is a _____ (acute, right, obtuse) angle.
- Angle JKL is a _____ (number) degree angle.

A-B-C-D-E-F-G-H-I
Ei bi si di l ef yi eich ai

J-K-L-M-N-O-P-Q-R-
Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z
es ti iu vi dabliu ex uay zi

Instructions: Describe the angle using mathematical language.



Useful Language

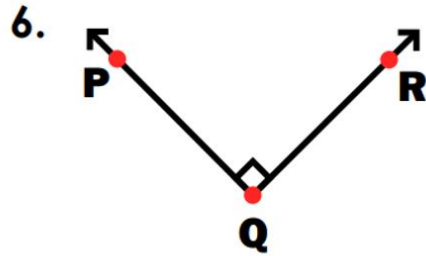
- Angle MNO is a _____ (acute, right, obtuse) angle.
- Angle MNO is a _____ (number) degree angle.

A-B-C-D-E-F-G-H-I
Ei bi si di l ef yi eich ai

J-K-L-M-N-O-P-Q-R-
Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z
es ti iu vi dabliu ex uay zi

Instructions: Describe the angle using mathematical language.



Useful Language

- Angle PQR is a _____ (acute, right, obtuse) angle.
- Angle PQR is a _____ (number) degree angle.

A-B-C-D-E-F-G-H-I

Ei bi si di l ef yi eich ai

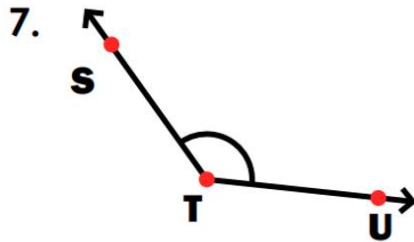
J-K-L-M-N-O-P-Q-R-

Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z

es ti iu vi dabliu ex uay zi

Instructions: Describe the angle using mathematical language.



Useful Language

- Angle STU is a _____ (acute, right, obtuse) angle.
- Angle STU is a _____ (number) degree angle.

A-B-C-D-E-F-G-H-I

Ei bi si di l ef yi eich ai

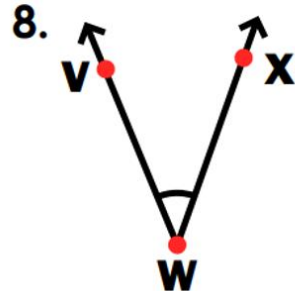
J-K-L-M-N-O-P-Q-R-

Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z

es ti iu vi dabliu ex uay zi

Instructions: Describe the angle using mathematical language.



Useful Language

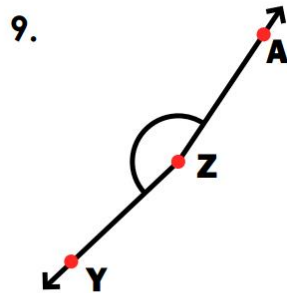
- Angle VWX is a _____ (acute, right, obtuse) angle.
- Angle VWX is a _____ (number) degree angle.

A-B-C-D-E-F-G-H-I
 Ei bi si di l ef yi eich ai

J-K-L-M-N-O-P-Q-R-
 Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z
 es ti iu vi dabliu ex uay zi

Instructions: Describe the angle using mathematical language.



Useful Language

- Angle AZY is a _____ (acute, right, obtuse) angle.
- Angle AZY is a _____ (number) degree angle.

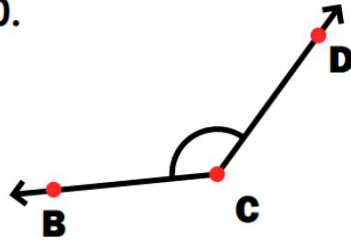
A-B-C-D-E-F-G-H-I
 Ei bi si di l ef yi eich ai

J-K-L-M-N-O-P-Q-R-
 Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z
 es ti iu vi dabliu ex uay zi

Instructions: Describe the angle using mathematical language.

10.



Useful Language

- Angle BCD is a _____ (acute, right, obtuse) angle.
- Angle BCD is a _____ (number) degree angle.

A-B-C-D-E-F-G-H-I

Ei bi si di l ef yi eich ai

J-K-L-M-N-O-P-Q-R-

Yei kei eol em en ou pi kiu ar

S-T-U-V--W--X-Y-Z

es ti iu vi dabliu ex uay zi

Lesson 10 – Handout #2

Handout 2 Angles & Degrees

Instructions.

1. Choose a role (student or math teacher).
2. Write a conversation according to the roles of each participant.
3. The teacher's role consists of thinking about the best way to teach a mathematics class related to angles and degrees.
4. The student's role consists of analyzing and asking questions related to angles and degrees to the teacher.
5. Use the useful language below and any relevant concepts (vocabulary) in English learned in previous tasks to help you construct your conversation.
6. Practice performing the conversation as many times as possible. Pay attention to the correct pronunciation of the words.

Useful language			
Asking for an explanation	Giving an explanation	Checking understanding	Other useful phrases
<ul style="list-style-type: none"> • Could you explain to me...? • I don't really understand...? • What do you mean by...? • I don't understand, I'm sorry. Would you mind explaining that again? 	<ul style="list-style-type: none"> • Look, it's like this... • What I mean is... • Let me explain... • What I am trying to say is... • In other words, ... 	<ul style="list-style-type: none"> • Does that mean...? • Do you mean...? • If I understand right... • Do I make myself clear? • Do you know what I mean? • Are you following me? 	<ul style="list-style-type: none"> • Hmm, let me think... • That's a good question... • Well, let's see... • I know/see what you mean • How interesting

Start writing your conversation here:

Lesson 10 – Handout #3

Handout 3

Instructions.

1. Read the passage Kinds of Angles.
2. Answer the questions below in complete form based on the reading.
3. Be prepared to share your answers with your classmates.

Kinds of Angles

Did you know that angles are measured with circles? An angle is a slice of a circle. Look at the two rays that are the arms of an angle. They come to a point where their endpoints meet. You can almost see it as a slice of pie. The whole pie, a full circle, has 360° . If we cut the circle up into 360 equal parts, each part would be one degree. We measure angles by how many of those tiny parts, or degrees, fit between the two arms of the angle. If an angle has exactly 90° it is called a right angle. Two perpendicular lines form a right angle. A right angle has a perfectly square corner, like we see on a square or a rectangle. If the angle has less than 90° it is called an acute angle. All acute angles are smaller than right angles. An acute angle fits inside the arms of a right angle if you placed the angles on top of each other. If the angle has more than 90° it is called an obtuse angle. All obtuse angles are bigger than right angles. A right angle could fit inside the arm of an obtuse angle. Did you know that even a straight line is an angle? It's called a straight angle. It measures 180° , exactly half of a circle. An angle that is so large that it bends past the straight line, but is still smaller than a full circle, is called reflex angle. It's fun to work angles!


Questions:

1. What shape is used to measure angles?
2. What unit is used to measure angles?
3. What kind of angles are a part of the squares and rectangles?
4. What is the difference between an acute angle and an obtuse angle?

Lesson 10 – Handout #4

Handout 4 Asynchronous Session: Triangles

Instructions

7. Watch the YouTube video about types of triangles ([here](#)) and read along the transcript provided below.
8. In the transcript, highlight in yellow five words that you find difficult to pronounce in English. Notice that the speaker in the video does not have a standard American English accent.
9. Look up your five words in the Oxford Learner's Dictionary ([here](#)). Listen carefully to the pronunciation of the words by clicking the pronunciation icon  and repeat.
10. In the table below, add your five words and include the link to the dictionary entry.

Word	Link to the dictionary entry
e.g. triangle	https://www.oxfordlearnersdictionaries.com/us/definition/english/triangle?q=triangle
1.	
2.	
3.	
4.	
5.	

Link to the YouTube video: <https://www.youtube.com/watch?v=NAxRP38GtzQ>

Link to the Oxfords Learner's Dictionary:

<https://www.oxfordlearnersdictionaries.com/us/>

Transcript:

In this video, we're going to look at the four types of triangle: equilateral triangles, isosceles triangles, right-angled triangles, and scalene triangles. And remember what makes them all triangles is that they all have three straight sides.

If we start with the equilateral triangle, the important thing about these is that all three sides are exactly the same length, and all three angles are 60 degrees. So if you need to check whether a triangle is equilateral, just take a look at the angles, or see if the sides have these little dashes on, which will tell you that they're all the same length.

Equilateral triangles also have three lines of symmetry, and the rotational symmetry of order three, which just means that there are three positions within a complete rotation that they look the same.

Next up is the isosceles triangle, whose key feature is that they have two sides that are the same length, and again you'll normally see the equal sides drawn with these little dashes on. They also have two angles that are the same size, one line of symmetry, and no rotational symmetry.

Moving on to right-angled triangles, the special thing about these is that they have a right angle, which they normally show by putting this small square in the corner. Right-angled

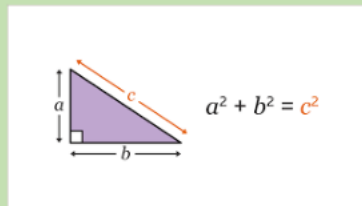
triangles don't normally have any lines of symmetry or any rotational symmetry. But if it happens to be a right-angled isosceles triangle, which is where it has a right angle and two sides that are the same length, then it will have one line of symmetry.

Last, we have the scalene triangle, and the special thing about these is that all three sides are different lengths, and so all three angles are different sizes. This means that they have no lines of symmetry and no rotational symmetry.

Anyway, that's everything for this video, so hope you enjoyed it and we'll see you again soon.

Triangles

Vocabulary and definitions



To participate in the activity:

I can try!

I'll do it!

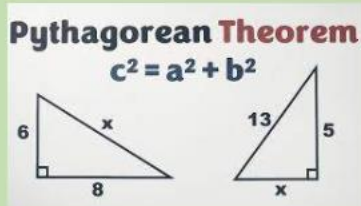
I have a question.

I don't know how to pronounce this word.

I think it is _____.

1. A _____ is a statement that has been proven to be true.

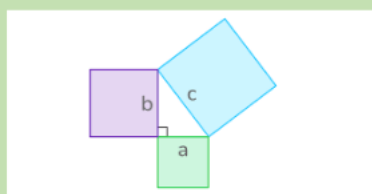




To participate in the activity:

I can try!
I'll do it!
I have a question.
I don't know how to pronounce this word.
I think it is_____.

2. The Pythagorean Theorem describes an important geometric _____ between the three _____ of a _____ triangle.

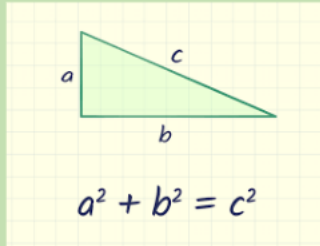


To participate in the activity:

I can try!
I'll do it!
I have a question.
I don't know how to pronounce this word.
I think it is_____.

3. To understand the Pythagorean Theorem you need to know about _____, _____, _____, and _____.

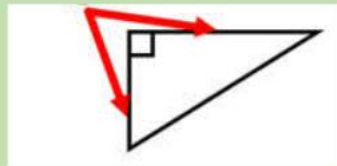




To participate in the activity:

I can try!
I'll do it!
I have a question.
I don't know how to pronounce this word.
I think it is_____.

4. The _____ is the longest side of a right triangle.



To participate in the activity:

I can try!
I'll do it!
I have a question.
I don't know how to pronounce this word.
I think it is_____.

5. The _____ of a triangle are the two opposite sides of the right angle.





To participate in the activity:

I can try!

I'll do it!

I have a question.

I don't know how to pronounce this word.

I think it is _____.

6. The Pythagorean Theorem is a useful tool that can help you to discover the _____.



Lesson 11 – Handout #2

Handout 2b

Task: Imagine that you are preparing to teach a mathematics class about the Pythagorean theorem. Since this is going to be the first time you teach mathematics in English, you asked a colleague to help you write a clear explanation of what the Pythagorean theorem is, what the Pythagorean theorem is useful for, and solve one Pythagorean equation to figure out the length of one side of a right triangle.

Useful language to talk to your classmate

What do you think?
Do you know what _____ means?
I don't remember how to say [*Spanish word*] in English.
How do you say [*Spanish word*] in English?
We can say that...
We can add that...
I am not sure...
I agree / I disagree

Guide to help you construct your explanation

- **First**, explain what the Pythagorean theorem is and for what type of triangles we use it for.

For example...

In this lesson, we're going to learn about The Pythagorean theorem. The Pythagorean theorem describes...

To understand the Pythagorean theorem, we need to know about...

The Pythagorean theorem doesn't apply to ALL triangles. It ONLY applies to...

The Pythagorean theorem goes like this: For a right triangle with legs 'a' and 'b' and hypotenuse 'c'... The hypotenuse is...

- **Then**, explain why the Pythagorean theorem is useful, what do we use it for?

For example...

The Pythagorean theorem is a useful tool that can help you...

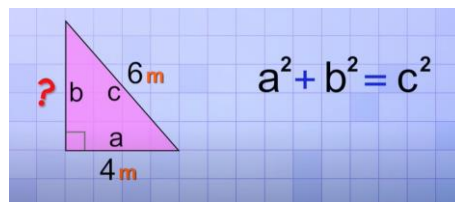
To use the Pythagorean theorem, you first need to...

- **Finally**, use the Pythagorean theorem to find the length of one of the legs (side 'b') of a right triangle. In other words, explain how we use the Pythagorean theorem to solve for the unknown value 'b'.

Side 'c' (hypotenuse): 6 m

Side 'a': 4 m

Side 'b': ??



For example...

Let's try one example. Imagine that you have a right triangle that...

To use the Pythagorean theorem to find the unknown value 'b', we first should... Then, we need to...

a right

Lesson 11 – Handout #3

Handout 3

Instructions:

Part I

Watch the video below about the pronunciation of the consonant sound / θ / as in "think" to learn how to pronounce this sound used in words like "math," "arithmetic," or "length."

['TH': Consonant Sound / θ / as in "think"- American English Pronunciation](#)

Part II

Practice with your classmates the pronunciation of the /θ/ consonant sound in following commonly used words and phrases in math. You can check the *Oxford Learners Dictionary* online for help: <https://www.oxfordlearnersdictionaries.com/us/>.

Pythagorean theorem

Pythagoras' theorem

in mathth

in ariththetic

mathth problems

ariththetic calculations

the main thing that you need to know

units of lengthth (cm, m, in)

the lengthth of the hypotenuse

the lengthths of the two legs

the lengthths of the two sides

the lengthth of the third side

the third side

the square root of third side

the ther side

the square root of th sides

Useful Language

- How do you pronounce this word?
- Are you sure that is the correct pronunciation?
- Say the word one more time.
- Remember to put your tongue between your teeth.
- I want to try...
- I am not sure...

Lesson 11 – Handout #4


Asynchronous Session: Trigonometry

Instructions

1. Watch the following video about trigonometry terminology.

Link to the video: <https://www.youtube.com/watch?v=gHcVGS8zxmU>

2. Complete the chart below with “the big six definitions” and their corresponding abbreviations.

3. Look up the six words in the Oxford Learner’s Dictionary ([here](#)). Listen carefully to the pronunciation of the words by clicking the pronunciation icon  and repeat.

Add the link to the dictionary entry in the chart below.

Link to Cambridge dictionary: <https://dictionary.cambridge.org/us/>

Terminology	Abbreviation	Link to the dictionary entry
1.		
2.		
3.		
4.		
5.		
6.		

Lesson 12 – Handout #1

Handout 1

Instructions:

Watch the video about basic trigonometry. Complete the following sentences with the missing words. Be prepared to share your answers with your classmates.

1. The six basic trigonometric functions involve _____ triangles.
2. _____ is just an angle that is not the right angle.
3. The longest side on the triangle is called the _____.
4. The other side, that is next to theta, that is not the hypotenuse is called the _____ side.
5. And the other side, the third side, directly opposite to theta, is the _____ side.
6. In the example, the $\sin \theta$ value is _____, the $\cos \theta$ value is _____, and the $\tan \theta$ value is _____.
7. In the example, the $\csc \theta$ value is _____, the $\sec \theta$ value is _____, and the $\cot \theta$ value is _____.

To check the exercise:

- I can try!
- I can read number...
- I want to answer number...
- I don't know how to pronounce this word...
- I think I heard...
- I am not sure...

Lesson 12 – Handout #2 (a)

Handout: Trigonometry

Task A: Finding Missing Sides

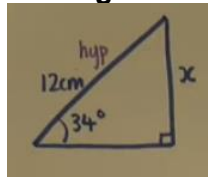
Instructions:

1. In pairs, watch the video and complete the script on how to find missing sides.
2. After that, practice the script aloud by taking turns with your partner.
3. Consult the dictionary to check pronunciation if necessary.

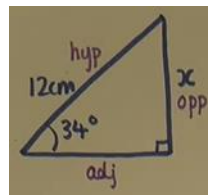
<https://www.oxfordlearnersdictionaries.com/us/>

Useful language to talk to your classmate	Word bank
I am missing the answer for number ____. Number __ is _____. Do you know what _____ means? I am not sure... I think it is... I don't remember how to pronounce this word. Let's check this word in the dictionary. I agree / I disagree Now it is your turn.	equation sine sides longest opposite hypotenuse adjacent angle opposite length

Script: Finding Missing Sides

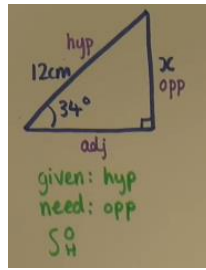


1. The first thing I need to do is to identify my _____ (1), my _____ (2), and my _____ (3). The hypotenuse has already been identify, it is the _____ (4) side. It is the diagonal opposite the right _____ (5).
2. I now need to identify what my other _____ (6) are. On the right, I have my side X that is opposite the angle, so that is my _____ (7) side. That means this side is the adjacent.

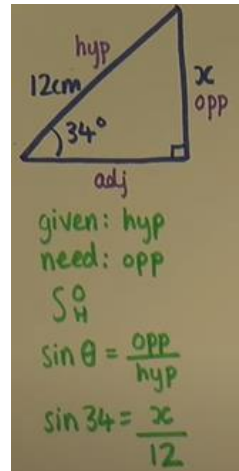


3. I have been given my hypotenuse and I have been asked to find my opposite. Using the hypotenuse and my opposite I can consult my equation or my acronym

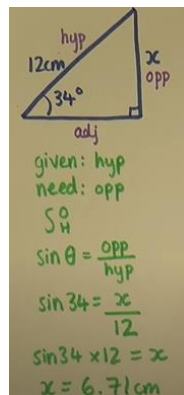
to find out which one of these I need to use. So I am using the hypotenuse and opposite, so it is the _____(8) the one I have to use.



4. So I am going to start filling in these values. So sine 34 degrees is equal to the opposite which we don't know, so I'm just going to put that as in x over the _____(9) of the hypotenuse which is 12.



5. Now I need to rearrange this _____(10) to give me X on its own. I need to multiply both sides by 12. That gives me $\sin 34 \times 12 = x$. You'll need a calculator for this. So $x = 6.71\text{cm}$.



Lesson 12 – Handout #2 (b)

Handout: Trigonometry

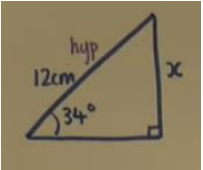
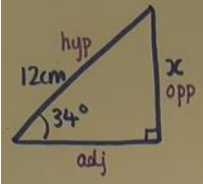
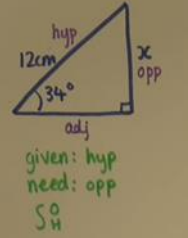
Task B: Finding Missing Sides

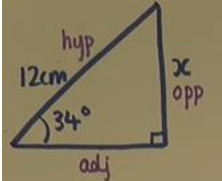
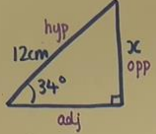
Instructions:

1. In pairs, watch the video on how to find missing sides.
2. After that, complete the chart by explaining each part of the procedure. You can use your own words or use the video as a resource to construct your explanation.
3. Consult the dictionary to check pronunciation if necessary.

<https://www.oxfordlearnersdictionaries.com/us/>

Useful language to talk to your classmate	Vocabulary and phrases
What do you think? Do you know what _____ means? I don't remember how to say [<i>Spanish word</i>] in English. How do you say [<i>Spanish word</i>] in English? We can say that... We can add that... I am not sure... I agree / I disagree	hypotenuse, opposite, adjacent the longest side right angle opposite the angle opposite side is equal to sine theta θ

	Explanation	Procedure
1	The first thing I need to do is to identify my hypotenuse, my opposite, and my adjacent. The hypotenuse is already been identify. It is the longest side opposite the right angle.	
2	Now I need to identify...	
3	Now I need to find out which equation I am going to use...	<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 5px; text-align: center;"> SO CA TO SH CH TA </div> <div style="text-align: center;">  </div> </div>

4	Then, I can start filling in these values...	 <p> hyp 12cm 34° adj x opp </p> <p> given: hyp need: opp S_H $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\sin 34 = \frac{x}{12}$ </p>
5	Finally, I need to rearrange this equation...	 <p> hyp 12cm 34° adj x opp </p> <p> given: hyp need: opp S_H $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\sin 34 = \frac{x}{12}$ $\sin 34 \times 12 = x$ $x = 6.71 \text{ cm}$ </p>

Lesson 12 – Handout #3

Stressed Syllabus		
Instructions: Pronounce the word aloud, notice where the word is stressed and drag it to the corresponding box. Consult the dictionary if necessary. https://www.oxfordlearnersdictionaries.com/us/		
Useful Language	Stressed in second syllable from the last	Stressed in third syllable from the last
<input type="checkbox"/> I think the word _____ has the stressed in the (second/third) syllable from the last.		
<input type="checkbox"/> Let's check this word in the dictionary.		
		Tangent
		Angle
		Perpendicular
		Equation
		Trigonometry
		Hypotenuse
		Adjacent
		Geometry
		Parallel
		Opposite
		Direction
		Equal

Lesson 12 – Handout #4

Handout 4 Asynchronous Session

I. **Instructions.** Complete the sentences below with the correct vocabulary words.

opposite	hypotenuse	trigonometry	perpendicular
cotangent	radius	parallel	adjacent

1. A _____ is a measure of distance from the center of any circular object to its edge.
2. The _____ number sentence $10 - 7 = 3$ will be $10 = 3 + 7$.
3. _____ concerns the functions of angles, such as sine, cosine and tangent.
4. Using the Pythagorean Theorem, the mathematician was able to find the triangle's _____.
5. A _____ line or surface points straight up, rather than being horizontal.
6. The angles XWY and YWZ are _____ to each other as they share a common vertex W and a common side YW.
7. _____ lines are in the same direction and never meet.
8. The _____ function is used to find the unknown angle or sides of a right triangle.

II. **Instructions.** Check the pronunciation of the following words in the dictionary. Then practice their pronunciation and record yourself. Upload your recording to Google Classroom.

Dictionary: <https://dictionary.cambridge.org/>

- ⇒ trigonometry
- ⇒ radius
- ⇒ adjacent
- ⇒ hypotenuse
- ⇒ value
- ⇒ sine (sin)
- ⇒ cosine (cos)

- ⇒ tangent (tan)
- ⇒ secant (sec)
- ⇒ cosecant (csc)
- ⇒ cotangent (cot)
- ⇒ theta θ
- ⇒ perpendicular
- ⇒ complementary
- ⇒ hypotenuse
- ⇒ opposite
- ⇒ parallel

Lesson 13 – Handout #1

TRIGONOMETRY

- Instructions: Listen to the audio and complete with the information requested.

Six main trigonometric functions:

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.

To participate in the activity:

I can try!

I'll do it!

I have a question.

I don't know how to pronounce this word.

I think it is _____.



TRIGONOMETRY

- Instructions: Listen to the audio and complete with the information requested.

- Trigonometric functions are important when _____ and modeling periodic phenomena.



To participate in the activity:

I can try!

I'll do it!

I have a question.

I don't know how to pronounce this word.

I think it is _____.

TRIGONOMETRY

- Instructions: Listen to the audio and complete with the information requested.
- The relationship between the trigonometric functions and the sides of a triangle are as follows:
- Sine $\theta = \frac{\quad}{\quad}$.
- $\theta = \frac{\quad}{\quad}$ / opposite.
- $\theta = \frac{\quad}{\quad}$ / adjacent.
- $\theta = \frac{\quad}{\quad}$ / hypotenuse.
- Tangent $\theta = \frac{\quad}{\quad}$.
- $\theta = \frac{\quad}{\quad}$ / $\frac{\quad}{\quad}$.

To participate in the activity:

I can try!

I'll do it!

I have a question.

I don't know how to pronounce this word.

I think it is _____.



Lesson 13 – Handout #2

Handout 2

Instructions:

Read aloud the words in the box below. Pay attention to the underlined sound in each word. List three words for every sound studied in this course. When you finish, practice pronouncing these words with your classmates.

three, exponent, obtuse, sum, revision, operation, arithmetic, reverse, acute, plus, conversion, fraction, addition, equation, multiplication, rule, surface, length

1	2	3	4	5	6
[ʃ] add <u>i</u> tion	[ʒ] div <u>i</u> sion	[ʌ] n <u>u</u> mber	[u] incl <u>u</u> de	[ɪ] simp <u>l</u> ify	[θ] math <u>θ</u>

To check the exercise:

- I can try!
- I can read column number...
- I don't know how to pronounce this word correctly.
- I am not sure about the pronunciation of this word.

Lesson 13 – Handout #3

Handout 3

Instructions

Describe the steps that we need to follow to solve the problems below using the order of operations rule (watch [this video](#) to refresh your memory). To describe each step, use one sequencing word. Remember that sequencing language allows us to arrange or organize actions and steps in the correct order.

Examples of sequencing words:

first, next, then, after that, last, finally, before, after

Order of Operations Guide

step 1: parenthesis (): FIRST, solve all problems in parenthesis.

step 2: exponents x^2 , x^3 , x^4 : NEXT, solve any numbers that have exponents.

step 3: multiply or divide: THEN, solve any multiplication or division problems (going from left to right).

step 4: add or subtract: FINALLY, solve any addition or subtraction problems (going from left to right).

Example 1: $(3^2 \times 4) + 6$

Step 1: FIRST, we do 3 to the power of 2, so we multiply 3 x 3, which is 9.

Step 2: THEN, we do the operation inside the parentheses BEFORE we do any operation outside parentheses. For this operation, we multiply 9 x 4, which equals 36.

Step 3: LAST, we add 36 + 6, which is 42.

Example 2: $10 \times (4 + 5)$

Step 1: FIRST, we add 4 + 5, which equals 9.

Step 2: THEN, we multiply 10 x 9, which is 90.

1. $18 - 36 \div 9 \times 4 + 8 =$
division

$18 - 4 \times 4 + 8 =$
multiplication

$18 - 16 + 8 =$
subtraction

$2 + 8 =$
addition

10

2. $5 \times [(4 + 2) + 6] - 21 \div 7 \times 3 =$
parentheses

$5 \times [6 + 6] - 21 \div 7 \times 3 =$
parentheses

$5 \times 12 - 21 \div 7 \times 3 =$
multiplication

$60 - 21 \div 7 \times 3 =$
division

$60 - 3 \times 3 =$
multiplication

$60 - 9 =$
subtraction

51

3. $3 \times (4^2 + 5^2) =$
exponents inside parentheses

$3 \times (16 + 25) =$
parentheses

$3 \times 41 =$
multiplication

123

4. $(5 + 7)^2 + 9 \times 3 =$
parentheses

$(12)^2 + 9 \times 3 =$
exponents

$144 + 9 \times 3 =$
multiplication

$144 + 27 =$
addition

171

Lesson 13 – Listening Quiz #3

Listening Quiz #3

Total points: 12 pts / 5% Points obtained: ___ pts. / ___ %

Time allotted: 20 min

Name:

General instructions: Watch the video three times and listen carefully. Choose the best answer to complete the following exercise. Once finished, submit your quiz to Google Classroom. You have 20 minutes to complete and submit this quiz.

Specific instructions: Listen to a mathematics professor talking about *trigonometry*. Fill in the blanks with the missing words from the box below.

hypotenuse	cosine (cos)	tangent (tan)	equations	opposite	cotangent (cot)
secant (sec)	length	cosecant (csc)	soh – cah - toa	adjacent	sine (sin)

1. One way that is easy to memorize the _____ is by using the phrase _____.
2. The easiest way to find the _____ is to go to the 90-degree angle and draw an arrow to the _____ side of that 90-degree angle.
3. The _____ side is always touching the angle but not the hypotenuse.
4. The _____ is equal to the opposite over the hypotenuse.
5. The _____ is equal to the adjacent over the hypotenuse.
6. The _____ is equal to the opposite over the adjacent.
7. To find the _____ of the angle a , what you have to do is to flip the sine (sin) of a .

8. To find a _____ of the angle a , what you have to do is to flip the cosine (cos) of a .
9. To find the _____ of the angle a , what you have to do is to flip the tangent (tan) of a .

Lesson 13 – Reading Quiz #3

University of Costa Rica
An Online English for Specific Purposes Course for Mathematics Professors
Instructors: Pamela Fallas, Marilyn Araya & Randall Montero

Reading Quiz #3

Total points: 8pts / 5% Points obtained: ___ pts. / ___ %

Time allotted: 20 min

General instructions: Read carefully and choose the best answer. Once finished, submit your quiz to Google Classroom. You have 20 minutes to complete and submit this quiz.

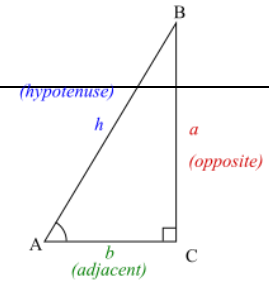
Specific instructions: Read the next passage. Answer the questions below.

Trigonometry

Trigonometry is one of the most important branches in mathematics that finds huge application in diverse fields. The branch called “Trigonometry” basically deals with the study of the relationship between the sides and angles of the right-angle triangle. Hence, it helps to find the missing or unknown angles or sides of a right triangle using the trigonometric formulas, functions or trigonometric identities. In trigonometry, the angles can be either measured in degrees or radians. Some of the most commonly used trigonometric angles for calculations are 0° , 30° , 45° , 60° and 90° . In trigonometry, there are six trigonometric functions that relate the angle measures of a right triangle to the length of its sides. They can be used to find the ratio of the side lengths. The six trigonometric functions are sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot). Explicitly, they are defined below as functions of the known angle A , where a , b and h refer to the lengths of the sides in the accompanying figure:

- **Sine** (denoted sin), defined as the ratio of the side opposite the angle to the hypotenuse.
- **Cosine** (denoted cos), defined as the ratio of the adjacent leg (the side of the triangle joining the angle to the right angle) to the hypotenuse.
- **Tangent** (denoted tan), defined as the ratio of the opposite leg to the adjacent leg. The hypotenuse is the side opposite to the 90-degree angle in a right triangle; it is the longest side of the triangle and one of the two sides adjacent to

angle A . The adjacent leg is the other side that is adjacent to angle A . The opposite side is the side that is opposite to angle A . The terms perpendicular and base are sometimes used for the opposite and adjacent sides respectively.



1. What is the definition of trigonometry?
2. How are angles measured in trigonometry?
3. What are the most commonly used trigonometric angles for calculations?
4. How are the six trigonometric functions used?
5. What are the six trigonometric functions?
6. How is sine (sin) defined?
7. How is tangent (tan) defined?
8. What is hypotenuse?



How do you pronounce these words?



To Participate in the Activity

- I can try!
- I'll do it!
- I do not remember how to pronounce this word, but I am going to try...
- I am not sure how to pronounce this word, but I want to try...

subtraction

add

addition

value

first

cotangent

Lesson 14 – Final Oral Exam

See Appendix G