Modeling the university drinking culture phenomenon

Alisson Serracín Morales[1], César Herrera Garro[1], and Fabio Sanchez[2]

[1] *Universidad de Costa Rica, Escuela de Matemática, San José, Costa Rica*
[2] *Universidad de Costa Rica, Escuela de Matemática-CIMPA, San José, Costa Rica*

Abstract

We study a susceptible-drinker-recovered (SDR) model for the drinking culture phenomenon in university atmospheres. We find conditions for this model to have extinction and endemic equilibrium points. We analyze different scenarios varying the basic reproductive number and initial conditions. Deterministic solutions are compared with stochastic simulations.

Keywords: Mathematical models, social modeling, alcoholism, drinking problems, university drinking culture

1 Introduction

According to [2], university students are involved in an environment that implicates a lot of stress, lack of comprehension from teachers, anxiety and other problems related to mental health, their socioeconomy situation, etc... These problems are often related with a high alcohol consumption, as said in [1], allowing the creation of a drinking culture among university students. This social phenomenon provokes a dynamic of nonlinear interactions between individuals in different stages.

In the past, deterministic epidemiological models have been used to study the dynamics of social phenomena such as drug addictions [3], alcoholism [7], social networks [10] etc... For example, in [7] they created a model to identify mechanisms that change the conversion of a population of nondrinkers to one of the drinkers, considering relapse. This model is similar to ours, with the difference that they don't include interaction between susceptible and recovered populations.

Besides the analysis we have done with our deterministic model, we have also studied a discrete time stochastic model. These kind of models have been studied using Markov Chains for either discrete time [12] or continuos time [13]. Infectious diseases as COVID-19 [14], HIV, Tuberculosis, and Hepatitis B [15] have been modeled using Markov chains. For our model implementation, we use Poisson processes, which are also used for epidemic models, such as models for COVID-19 [16, 17].

Our model's goal is to study the dynamic of the university drinking culture phenomenon. Quantifying phenomena like this helps to understand how they evolve in time, which could be helpful to design adequate policies to deal with them. Also, it is interesting to explore a stochastic model. It allows us to look at possible events that can't happen in a deterministic model. Often, less probable events happen, and deterministic models discard these events. For this reason, stochastic models help us to understand, in a different way, phenomena such as drinking culture.

2 Mathematical Model

2.1 Model's description

Table 1: Model's parameters an their definitions.

Parameters	Definition
β	transmission rate
μ	per-person departure rate from university environments
ϕ	per-person recovery rate
ρ	relapse rate
γ	rate of recovered that have a positive impact in the susceptible ones

Our model is based on the model constructed in [7], which explores interactions between individuals in shared drinking environments.

Our population N is all the people that are active students in any university in Costa Rica. Students who have more than three alcoholic beverages on one occasion, on a frequency of zero to two monthly, enter the susceptible category. For the second category, students who have more than three alcoholic beverages on one occasion, on a frequency that exceeds zero to two monthly, enter the "infected" class, known as alcoholic students. At last, students who were in the infected class and nowadays do not classify on it fall into the recovered category.

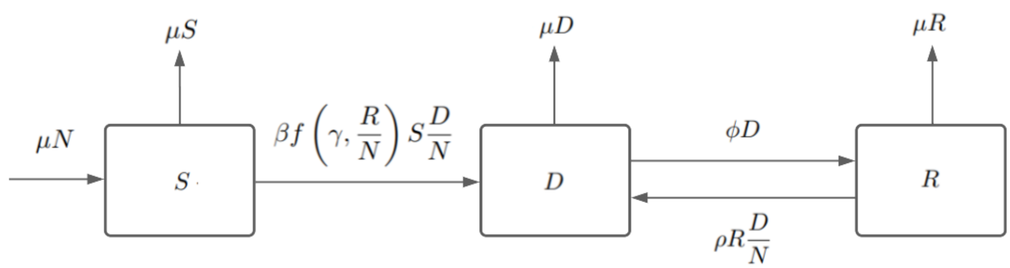


Figure 1: Model's flow chart.

Our model allows us to study reinfection, known as relapse, in the case of the drinking culture phenomenon. Also, the model differs from the presented in [7] because it considers new interactions between the susceptible and recovered population. This is represented by the function

$$f\left(\gamma, \frac{R}{N}\right) = \frac{1}{1+\gamma\frac{R}{N}}.$$

Here, f represents a positive impact that some of the recovered students might have on the susceptible ones, leading to a minor percentage of susceptible students falling into the drinker's category. For example, a recovered student giving good advice about moderate alcohol consumption to a susceptible one contributes to f, and it's related to the parameter γ that we already defined.

The model contemplates a system through which students enter and leave out the student community.

The following is the system of nonlinear differential equations:

$$\frac{dS}{dt} = \mu N - \beta f\left(\gamma, \frac{R}{N}\right) S\frac{D}{N} - \mu S, \tag{1}$$

$$\frac{dD}{dt} = \beta f\left(\gamma, \frac{R}{N}\right) S\frac{D}{N} + \rho R\frac{D}{N} - (\mu + \phi) D, \tag{2}$$

$$\frac{dR}{dt} = \phi D - \rho R\frac{D}{N} - \mu R \tag{3}$$

Also, for the system we have that

$$N = S + D + R \tag{4}$$

2.2 Estimated Parameters

Table 2: Model's parameters and their values.

Parameter	Values
β	[0,1]
μ	$\frac{1}{312}$
ϕ	0.17
ρ	0.42
γ	0.5
N	47500

The values of the parameters we use in later simulations are shown in Table 2. For obtaining the values of γ, ϕ, and ρ, we applied a survey with questions related to the university drinking culture phenomenon to a total of 450 students in Costa Rican universities. Meanwhile, to estimate μ, we based our criterion on the data shown in [8], of how many years a student last to finish their career. The parameter β is not fixed, and we are going to take it as $\beta \in [0, 1]$ in our study. N is taken based on official data from [9].

2.3 Dynamical Analysis

2.3.1 Basic Reproductive Number

Theorem 1. *The basic reproductive number is*

$$R_0 = \frac{\beta}{\mu + \phi} \tag{5}$$

See proof of Theorem 2 to see the proof of this theorem.

2.3.2 Drinking-free Equilibrium

The drinking-free equilibrium of the system is

$$(S_0, D_0, R_0) = (N, 0, 0) \tag{6}$$

that is, the state where a drinking culture does not exist.

Theorem 2. *The drinkers-free equilibrium is locally stable if and only if* $R_0 = \frac{\beta}{\mu+\phi} < 1$.

Proof. The drinkers-free equilibrium occurs when $(S_0, D_0, R_0) = (N, 0, 0)$. Using the technique of Linearization, we can find the Jacobian of the system:

$$J(S, D, R) = \begin{bmatrix} \frac{-\beta}{(N+\alpha R)}D - \mu & \frac{-\beta}{(N+\alpha R)}S & \frac{-\beta\gamma}{(N+\alpha R)^2}SD \\ \frac{\beta}{(N+\alpha R)}D & \frac{\beta}{(N+\alpha R)}S + \frac{\rho}{N}R - (\mu+\phi) & \frac{\beta\gamma}{(N+\alpha R)^2}SD + \frac{\rho}{N}D \\ 0 & \phi - \frac{\rho}{N}R & \frac{\rho D}{N} - \mu \end{bmatrix}$$

The Jacobian evaluated at the drinkers-free equilibrium yields:

$$J(N, 0, 0) = \begin{bmatrix} -\mu & -\beta & 0 \\ 0 & \beta - (\mu+\phi) & 0 \\ 0 & \phi & -\mu \end{bmatrix}$$

Observe that the eigenvalues of the Jacobian matrix are:

$$\lambda_1,\ \lambda_2 = -\mu,$$
$$\lambda_3 = \beta - (\mu + \phi)$$

The equilibrium is locally stable if all of the eigenvalues are negative. First, note that $\mu > 0$ and then we have that $\lambda_1 = \lambda_2 < 0$. Now, in order to ensure that $\lambda_3 < 0$, we must have that:

$$R_0 = \frac{\beta}{\mu + \phi} < 1.$$

Therefore, if the drinkers-free equilibrium is stable, it must be that $R_0 < 1$. □

2.3.3 Endemic equilibrium

Theorem 3. *The existence of three endemic-equilibrium points depends on the following three conditions:*

$$\begin{aligned} 1 &< \frac{\beta}{\mu + \phi} = R_0 \\ \gamma &< (\frac{1}{\mu} + \frac{1}{\phi})(R_0(\rho - \mu) - \beta - \mu R_\rho) \\ \gamma &> \frac{1}{\phi}(R_0(2\rho - \mu - \phi) - R_\rho(\phi + 2\mu)) \end{aligned}$$

Proof. Endemic-equilibrium points (S^*, D^*, R^*) are solutions of the cubic equation:

$$p(D) = aD^3 + bD^2 + cD + d$$

where:

$$d = N^3\mu^2(-\beta + \mu + \phi)$$
$$c = N^2\mu(\beta(\mu - 2\rho + \phi) + \phi(\rho + \gamma\phi) + \mu(2\rho + \gamma\phi))$$
$$b = N\rho(\beta(2\mu - \rho + \phi) + \mu(\rho + \ \gamma]\phi))$$

$$a = \beta\rho^2$$

For those solutions to be real and positive, i.e., $(S^* > 0, D^* > 0, R^* > 0)$ according to [5] we require:

$$\Delta = 18abcd – 4b^3d + b^2c^2 – 4ac^3 – 27a^2d^2 > 0.$$

Even more, according to [6], the number of positive roots of the polynomial p is either equal to the number of sign changes between consecutive (nonzero) coefficients, or is less than it by an even number. But, also, the number of negative roots is the number of sign changes after multiplying the coefficients of odd-power terms by -1, or fewer than it by an even number.

We know that $a = \beta\rho^2 > 0$, so according to the Descartes Rule of Signs a we must have that

$$d = N^3\mu^2(-\beta + \mu + \phi) < 0$$

$$c = N^2\mu(\beta(\mu - 2\rho + \phi) + \phi(\rho + \gamma\phi) + \mu(2\rho + \gamma\phi)) > 0$$

$$b = N\rho(\beta(2\mu - \rho + \phi) + \mu(\rho + \gamma\phi)) < 0$$

- $N^3\mu^2(-\beta + \mu + \phi) < 0$

$$N^3\mu^2(-\beta + \mu + \phi) < 0$$
$$\Leftrightarrow -\beta + \mu + \phi < 0$$
$$\Leftrightarrow \mu + \phi < \beta$$
$$\Leftrightarrow 1 < \frac{\beta}{\mu + \phi}$$
$$\Leftrightarrow 1 < R_0$$

- $N^2\mu(\beta(\mu - 2\rho + \phi) + \phi(\rho + \gamma\phi) + \mu(2\rho + \gamma\phi)) > 0$

$$N^2\mu(\beta(\mu - 2\rho + \phi) + \phi(\rho + \gamma\phi) + \mu(2\rho + \gamma\phi)) > 0$$
$$\Leftrightarrow \beta(\mu - 2\rho + \phi) + \phi(\rho + \gamma\phi) + \mu(2\rho + \gamma\phi) > 0$$
$$\Leftrightarrow \beta\mu - 2\beta\rho + \beta\phi + \phi\rho + \gamma\phi^2 + 2\mu\rho + \mu\gamma\phi > 0$$
$$\Leftrightarrow \beta(\mu - 2\rho + \phi) + \rho(\phi + 2\mu) > -\gamma\phi(\mu + \phi)$$
$$\Leftrightarrow \frac{\beta}{\phi(\mu + \phi)}(2\rho - \mu - \phi) - \frac{\rho}{\phi(\mu + \phi)}(\phi + 2\mu) < \gamma$$
$$\Leftrightarrow \frac{1}{\phi}(R_0(2\rho - \mu - \phi) - R_\rho(\phi + 2\mu)) < \gamma$$

- $N\rho(\beta(2\mu - \rho + \phi) + \mu(\rho + \gamma\phi)) < 0$

$$N\rho(\beta(2\mu - \rho + \phi) + \mu(\rho + \gamma\phi)) < 0$$
$$\Leftrightarrow \beta(2\mu - \rho + \phi) + \mu(\rho + \gamma\phi) < 0$$
$$\Leftrightarrow 2\beta\mu - \beta\rho + \beta\phi + \mu\rho + \mu\gamma\phi < 0$$
$$\Leftrightarrow \beta\mu - \beta\rho + \beta\mu + \beta\phi + \mu\rho < -\mu\gamma\phi$$
$$\Leftrightarrow \beta(\mu - \rho) + \beta(\mu + \phi) + \mu\rho < -\mu\gamma\phi$$
$$\Leftrightarrow (\mu + \phi)(\beta\frac{(\mu - \rho)}{\mu + \phi} + \beta + \frac{(\mu\rho)}{\mu + \phi}) < -\mu\gamma\phi$$

$$\Leftrightarrow (\frac{1}{\phi} + \frac{1}{\mu})(R_0(\rho - \mu) - \beta - \mu R_\rho) > \gamma$$

Therefore, for having three endemic-equilibrium points we need:

$$\begin{aligned} 1 &< \frac{\beta}{\mu + \phi} = R_0 \\ \gamma &< (\frac{1}{\mu} + \frac{1}{\phi})(R_0(\rho - \mu) - \beta - \mu R_\rho) \\ \gamma &> \frac{1}{\phi}(R_0(2\rho - \mu - \phi) - R_\rho(\phi + 2\mu)) \end{aligned}$$

□

2.3.4 Backward Bifurcation

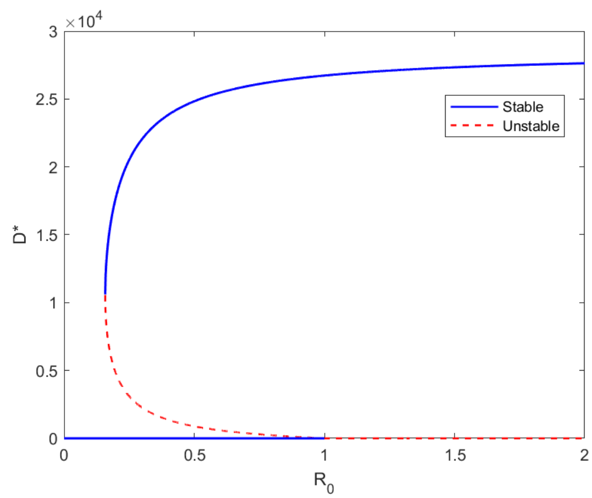


Figure 2: Backward Bifurcation

In this case, we plot R_0 vs infected population with $\mu = \frac{1}{312}$, $\phi = 0.17$, $\rho = 0.42$, $\gamma = 0.5$, $N = 47500$ and varying β. As shown, the system has a backward bifurcation. According to [4] a backward bifurcation has the characteristic of having a stable endemic equilibrium co-existing with a stable DFE when $R_0 < 1$. That means, $R_0 < 1$ is a necessary but not sufficient condition for a disease control. Actually, in order to ensure extinction, we need R_0 to be smaller than some value $R_c < 1$. Meanwhile, for $R_0 > 1$ the disease is likely to invade to a relatively high endemic level.

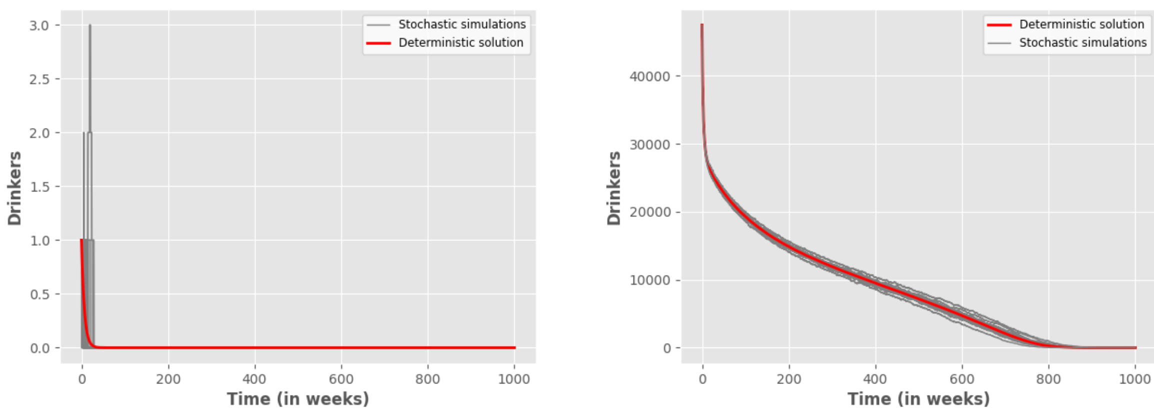
3 Results

We ran simulations using parameters shown in Table 2 and varying β. So that; we get different values for R_0. For initial conditions, we only vary $D(0)$, and we take $R(0) = 0$ and $S(0) = N - D(0)$. We separate the results in three cases depending on the value of R_0: $R_0 < R_c$, $R_c < R_0 < 1$ and $1 < R_0$.

We compare the deterministic solutions with stochastic simulations. For the stochastic model, we used Python for the implementation. We implemented a discrete time stochastic model in which we defined one random variable with Poisson distribution per event (entry, departure, infection, relapse, recovery). For every set of initial conditions and R_0 values selected, we ran 25,000 stochastic simulations, and we got the percentage of them that reached 0 drinkers and an endemic equilibrium after 1,000 weeks.

3.1 Case: when $R_0 < R_c$

When $R_0 < R_c$, we got that deterministic solution and all stochastic simulations reached a DFE. In Figure 3, we show two plots when $\beta = 0.02$ and so, $R_0 \approx 0.12$.



(a) $D(0) = 1$ (b) $D(0) = N$

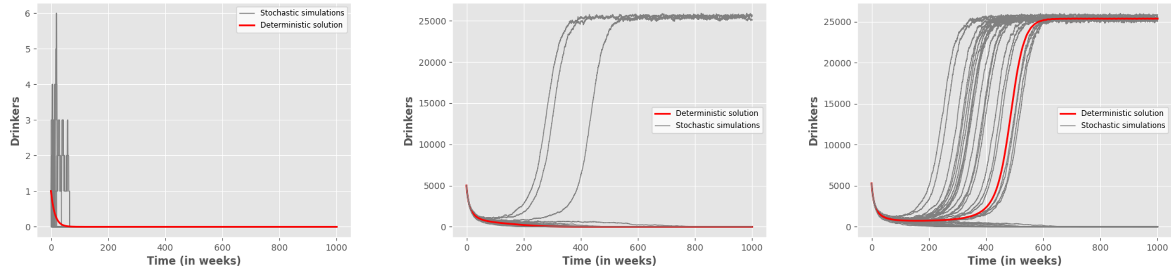
Figure 3: Deterministic model solution and stochastic simulations with $R_0 \approx 0.12$ for different initial conditions.

3.2 Case: when $R_c < R_0 < 1$

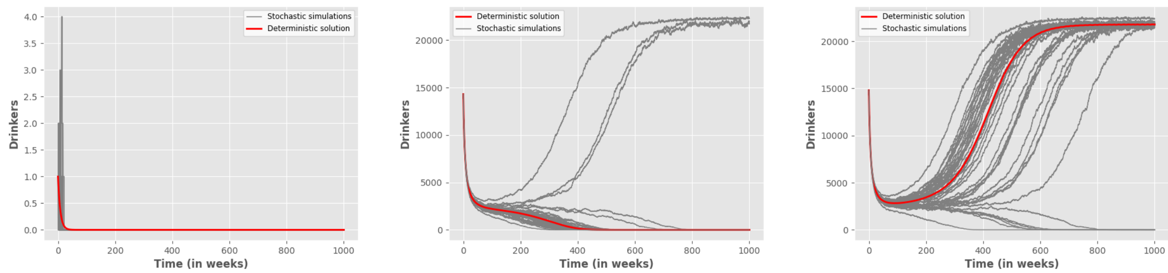
For $R_c < R_0 < 1$, we selected $\beta = 0.1$ and $\beta = 0.05$ to get $R_0 \approx 0.58$ and $R_0 \approx 0.29$, respectively. In both cases, when, $D(0) = 1$, we got that deterministic solution and 100% of the stochastic simulations went to zero drinkers eventually. However, these results change when we vary the initial conditions. For the case $R_0 \approx 0.58$, we got that a 4.4% and a 53.0% of the stochastic simulations reached an endemic equilibrium for $D(0) = 5000$ and $D(0) = 5300$, respectively. For the case $R_0 \approx 0.29$, we obtained that a 6.8% and a 93.2% of the stochastic simulations reached an endemic equilibrium for $D(0) = 14300$ and $D(0) = 14800$, respectively. In Figure 4, we show plots of these experiments.

3.3 Case: when $R_0 > 1$

When $R_0 > 1$, we know that deterministic solution goes to an endemic equilibrium with $D(0) = 1$. But, for the stochastic model, we got that, depending on the R_0, a high percentage of the stochastic simulations goes to a DFE. In particular, when $\beta = 0.3$ ($R_0 \approx 1.73$), 57.7% of the stochastic simulations went to a DFE. When $\beta = 0.2$ ($R_0 \approx 1.15$), 86.4% of the stochastic simulations went to a DFE. Plots of this simulations are shown in Figure 5.

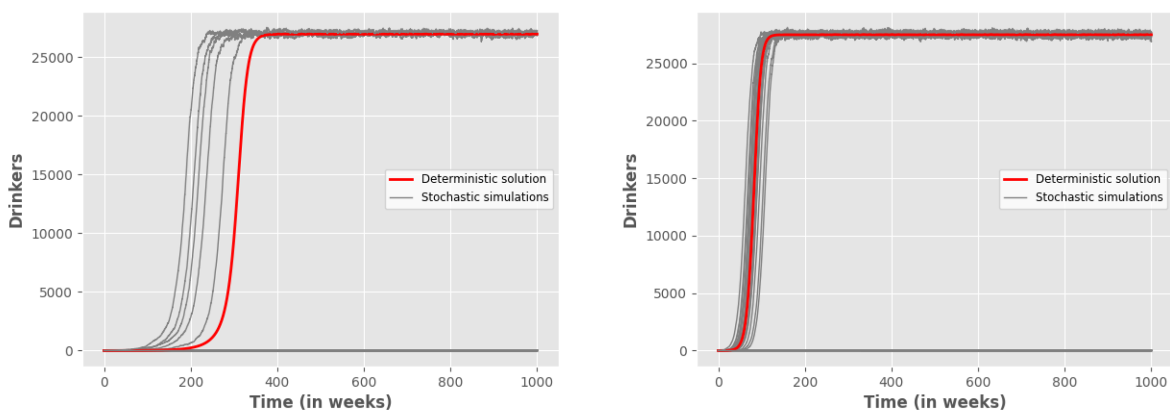


(a) $D(0) = 1, R_0 \approx 0.58$ (b) $D(0) = 5000, R_0 \approx 0.58$ (c) $D(0) = 5300, R_0 \approx 0.58$



(d) $D(0) = 1, R_0 \approx 0.29$ (e) $D(0) = 14300, R_0 \approx 0.29$ (f) $D(0) = 14800, R_0 \approx 0.29$

Figure 4: Deterministic model solution and stochastic simulations with different values of R_0, for different initial conditions.



(a) $R_0 \approx 1.15$ (b) $R_0 \approx 1.73$

Figure 5: Deterministic model solution and stochastic simulations with initial condition $D(0) = 1$ for different values of R_0.

4 Discussion

We proposed a susceptible-drinker-recovered (SDR) model to study the dynamic of the university drinking culture phenomenon, considering relapse and interaction between susceptible and recovered students. Afterward, we estimated γ, μ, ϕ and ρ as said in Section 2.2 and varied β from 0 to 1.

We found the basic reproductive number and conditions for the existence of endemic equilibrium points. There could be up to three endemic equilibrium points. However, for our estimated parameters, we could get up to two endemic equilibrium points depending on the value of the transmission rate. We found that our system has a backward bifurcation, and this implies that $R_0 < 1$ is not a sufficient condition to get extinction. As a result, in the deterministic model, we need a relatively small value for the transmission rate to guarantee extinction.

Related to the results shown in Section 3, when $R_0 < R_c$, we took $R_0 = 0.12$ and initial conditions $D(0) = 1$ and $D(0) = N$. In both cases, the deterministic solution and all stochastic simulations reached a DFE. For $R_0 > 1$, the deterministic solution tells us that, eventually, the drinker's population is going to reach an endemic equilibrium point. But, according to the stochastic simulations, it seems that a considerable percentage goes to a DFE. The most interesting case is when $R_c < R_0 < 1$. Here, we took $R_0 = 0.29$ and $R_0 = 0.58$. For both R_0, we ran simulations with $D(0) = 1$, which wasn't an interesting case because the deterministic solution and all stochastic simulations reached a DFE. But, for $R_0 = 0.29$ and $R_0 = 0.58$ with $D(0) = 14300$ $D(0) = 5000$ respectively, a tiny percentage of the simulations went to an endemic equilibrium, but the deterministic solution went to zero drinkers eventually. From this, we can conclude that for $R_0 < R_c$, one can see a few scenarios that we would have discarded just using the deterministic model.

One of the most unusual cases was for $R_0 = 0.58$ and $D(0) = 5300$. Here, the deterministic solution goes to an endemic equilibrium, but 47% of the stochastic simulations went to a DFE. At last, for $R_0 > 1$, we took $R_0 = 1.73$ and $R_0 = 1.15$ with $D(0) = 1$. For these cases, the deterministic model goes to an endemic equilibrium, but 57.7% and 86.4% respectively, of the stochastic simulations went to a DFE. So, it is shown that for some situations, depending on the R_0, the use of a stochastic model would give a bigger vision of the panorama, and all the possible scenarios that we could confront.

This work gives us a better understanding of the drinking culture phenomenon. However, we remark that our studied population was university students in Costa Rica. So, it would be interesting to study another kind of population and see how different the parameters and the model would be. Also, this model could be applied to comprehend the behavior of the consumption of other kinds of drugs. It is relevant to keep up with the evolution of this type of phenomenon to establish sound policies.

Acknowledgments

We thank all the people who participated in the survey for estimating parameters. We would also like to thank Universidad de Costa Rica, the institution that made this work possible.

References

[1] H.C. Becker. Effects of alcohol dependence and withdrawal on stress responsiveness and alcohol consumption. *Alcohol Res.*, 34(4):448–458, 2012.

[2] M. Cordero. Preocupación por sobrecarga académica y por mantener becas agudiza crisis de salud mental en estudiantes. Semanario Universidad. *https://tinyurl.com/ycksxrtp*

[3] D.A. Behrens, J.P. Caulkins, G. Tragler, J.L. Haunschmied, G. Feichtinger. A dynamic model of drug initiation: implications for treatment and drug control. *Math. Biosci.*, 159:1–20, 1999.

[4] A.B. Gumel. Causes of backward bifurcations in some epidemiological models. *Journal of Mathematical Analysis and Applications*, 395:355–365, 2012.

[5] P. Garret. Discriminant of cubics. Recovered from *https://www-users.cse.umn.edu/~garrett/m/mfms/notes_2013-14/03b_cubic_discriminant.pdf*

[6] W.L. Hosch. Descartes's rule of signs. Recovered from `https://www.britannica.com/science/Descartess-rule-of-signs`. *Encyclopedia Britannica*, 2011.

[7] F. Sanchez, X. Wang, C. Castillo-Chavez, D. Gorman, P. Gruenewald. 16 - Drinking as an Epidemic—A Simple Mathematical Model with Recovery and Relapse. *Academic Press*, 353–368, 2007.

[8] Programa Estado de la Nación. Octavo Estado de la Educación 2021. Recovered from *https://estadonacion.or.cr/wp-content/uploads/2021/09/Educacion_WEB.pdf*

[9] Universidad de Costa Rica. UCR en cifras. Recovered from *https://www.ucr.ac.cr/acerca-u/ucr-en-cifras.html*

[10] W. Pearson, M. Mccartney. Dynamics of Social Networks: a Deterministic Approach. *Int. J. Appl. Math. Comp. Sci.*, 12:545–551, 2002.

[11] P. van den Driessche, J. Watmough. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences*, 180:29–48, 2002.

[12] L.J.S. Allen, A.M. Burgin. Comparison of deterministic and stochastic SIS and SIR models in discrete time. *Mathematical Biosciences*, 163:1–33, 2000.

[13] L.J.S. Allen. A primer on stochastic epidemic models: Formulation, numerical simulation, and analysis. *Infectious Disease Modelling*, 2:128–142,2017.

[14] Z. Xu, H. Zhang, Z. Huang. A Continuous Markov-Chain Model for the Simulation of COVID-19 Epidemic Dynamics. *Biology (Basel)*, 11(2):190, 2022.

[15] C. Twumasi, Louis. Asiedu, Ezekiel. Nortey. Markov Chain Modeling of HIV, Tuberculosis, and Hepatitis B Transmission in Ghana. *Interdisciplinary Perspectives on Infectious Diseases*, 2019, 2019.

[16] S. Hussain, A. Zeb, A. Rasheed, T. Saeed. Stochastic mathematical model for the spread and control of Corona virus. *Advances in Difference Equations*, 2020:574, 2020.

[17] H. Ben Hassen, A. Elaoud, N. Ben Salah, A. Masmoudi. A SIR-Poisson Model for COVID-19: Evolution and Transmission Inference in the Maghreb Central Regions. *Arabian journal for science and engineering*, 46:93–102, 2021.