

# Pavement Deterioration

## Bayesian Regression in Pavement Deterioration Modeling: Revisiting the AASHO Road Test Rut Depth Model

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### Abstract

*Traditional pavement deterioration modeling is normally based on historical condition data alone without incorporating the fundamental relationships between the causal factors and the response. Also, typical approaches do not quantify the uncertainty of the predicted response. This paper uses Bayesian regression for pavement deterioration modeling. This method is applied to an existing model for the prediction of rut depth progression from the AASHO Road Test. A classical regression model developed elsewhere is herein summarized and its results are then compared with those from the Bayesian regression in order to validate. A second model based on the entire dataset of the AASHO road test is used to demonstrate the advantages of such approach. The models are capable of employing expert criteria combined with historical knowledge and current observations in order estimate posterior probabilistic distributions for the regression coefficients of the mechanistic equation. The predictive model calibrated to local conditions is able to forecast within pre-specified confidence intervals the range of values for the expected deterioration. Bayesian regression modeling produces more reliable predictions for deterioration performance, which in turn, can be used to improve decision-making on road management systems.*

**Keywords:** Pavement, performance, deterioration, Bayesian.

### Resumen

Tradicionalmente el modelaje del deterioro de pavimentos se basa en observaciones históricas de condición que no incorporan las relaciones fundamentales entre los factores causales y la respuesta. Adicionalmente dichos métodos no cuantifican el nivel de incertidumbre asociado con la respuesta predecida. Esta investigación utiliza regresión Bayesiana para modelar el desempeño de pavimentos. La metodología fue aplicada a un modelo existente que predice la progresión de deformaciones permanentes del experimento de la Asociación Americana de Oficiales de Autopistas y Transportes (AASHTO por sus siglas en inglés). Un modelo de regresión desarrollado anteriormente es resumido y utilizado para validar los resultados de la regresión Bayesiana. Un segundo modelo basado en la totalidad de la base de datos del experimento de la AASHTO se utiliza para demostrar las ventajas del método propuesto. Ambos modelos son capaces de emplear opiniones de expertos combinadas con conocimiento histórico y observaciones actuales con el fin de estimar distribuciones probabilísticas de los coeficientes de la ecuación mecanística. El modelo de predicción así calibrado a condiciones locales es capaz de predecir, dentro de rangos de confianza, valores de deterioro esperado. La regresión Bayesiana produce predicciones más confiables del deterioro y mejora la toma en sistemas de gestión vial.

**Palabras clave:** Pavimento, rendimiento, deterioro, Bayesian.

### Introduction

Pavement deterioration modeling can be grouped in basic types ranging from probabilistic and mechanistic to empirical (1). Markov Chain is perhaps the most popular probabilistic modeling technique. This technique has been explained by others (2, 3). The development of a Transition Probabilities matrix (TPM) has been the target of several papers (4, 5). Such mechanism captures the uncertainty of the response as one predicts future values of any response. The TPM is normally based in historical observations of how condition has evolved. Initial condition vectors (ICV) are multiplied  $n$  times by the matrix to obtain future conditions with immediate values after a survey concentrated in one cell of the ICV. Major drawbacks of such approach are: (a) it does not have a memory of the previous conditions (except the year immediate before), (b) the TPM is only valid for a similar repetition of the same levels of casual factors from the observations that create the TPM, hence depending on the assumption of “ceteris paribus”, and (c) it can not be used in other locations or under different conditions (i.e., levels of traffic, environmental exposure, pavement materials or structural deficiencies) because the model does not explicitly consider how the predictors affect the response (unless the TPM is calibrated for the new conditions).

Although purely mechanistic methods do provide a relationship between causal factors and a given response, their applicability is limited depending on the observed levels of the casual factors while developing the mechanistic equation (6). Such model fails in providing a measure of uncertainty, and mechanisms to calibrate the model parameters to local conditions. Also there are no guarantees that all predictors can be incorporated in the model. The range of applicability beyond the specific conditions of the empirical study is normally unclear, and it is difficult to adjust the model after new data becomes

available. Another drawback on mechanistic modeling is that the casual factors are normally given in terms of physical properties such as: pavement layer moduli, stresses, strains etc., rather than in explicit measures of distresses widely collected in pavement management such as roughness or deflections. Predictions from any deterministic or mechanistic model only provide the mean value, and are not capable of modeling the dispersion. The use of probabilistic approaches to address such limitations on performance modeling has been suggested elsewhere (7).

Empirical modeling is traditionally done by employing a regression technique to fits observations of a response to causal factors. Traditional regression modeling employs either: the least square or the maximum likelihood approach, to find the values of the coefficients of a given functional form (linear, exponential, potential, logarithmic or polynomial). According to (8), approaches such as maximum likelihood can work well with “models with few predictors” and large datasets. However, perfect separation and/or co-linearity issues between several casual factors may arise; producing bad estimation of the regression coefficients because of interactions between some predictors. A non linear transformation (i.e. exponential) is typically used to address this issue providing separation of the predictors. Other general concerns of regression techniques are over-fitting and model complexity (9). The use of Bayesian statistics (a form of penalized-likelihood) overcomes these difficulties (8).

## Objective

The objective of this paper is to present the basis and development of reliability based modeling employing Bayesian regression methodology.

## Methodology

### **Introduction to Bayesian Statistics**

In Bayesian statistics, the estimation of the result before any information (from trials, experiments or field data collection) is available and is known as the prior, and is regarded as the value an individual expects for some outcome before seeing any evidence (data). The evidence (observations) from a certain number of trials is regarded as the likelihood  $[P(\text{data}/\theta)]$ , and it expresses the result of a certain number of trails or experiments. Bayes theorem works as a mechanism for generating a posterior of any parameter mixing the

prior knowledge with the likelihood, the denominator is regarded in literature as the normalization or marginal term (Equation 1). Posterior distributions are tighter (less disperse) than priors or likelihoods alone, therefore posterior takes advantage of both to obtain a better distribution.

$$P(\theta / \text{Data}) = \frac{P(\text{data} / \theta) \times P(\theta)}{[P(\text{data} / \theta) \times P(\theta)] \times d\theta} \quad [1]$$

Bayesian statistics is currently combined with sampling techniques. The whole idea of sampling is to approach the solution of a mathematical problem by observing results from a large set of trials, instead of looking for a close form solution. Then, when one takes a large amount of trials or samples (as  $n$  tends to infinitum), the law of large numbers apply and, one can assure that the results of sampling will approximate the true value of the unknown. In brief, Bayesian statistics is a method that can learn from experience (observed data) and consider human opinions as a part of the analysis. The approach recognizes the existence of uncertainty in the predictors (and unobserved effects), hence the model takes advantage of simulation and conditional probabilities to produce a probabilistic representation of the model parameters that best fit to the observed data. This production of posterior probabilistic distribution for the parameters is equivalent to calibrating a regression model to local conditions, also regarded as the training phase (9). Such calibrated model can be used to predict responses for a new set of predictors.

Bayesian inference has taken advantage of Markov Chain Monte Carlo for solving complex problems (11). The simulation is done through the Metropolis-Hasting algorithm or its simplified form: the Gibbs algorithm. Results are given after the convergence of the MCMC chain(s). Different mechanisms are used for convergence such as observing various chains with good mixture and stable values of the posterior; also called the stationary state (12). This type of modeling does not assume that the response is concentrated in a point estimate as the mechanistic or deterministic modeling do. Bayesian regression modeling addresses the problems of the deterministic and mechanistic approaches by providing an updatable mechanism for mixing expert criteria with available observations to produce a locally calibrated model capable of representing uncertainty in the coefficients and in the response.

## Bayesian Regression

Bayesian regression is somehow similar to traditional regression in the sense that a functional form (equation) relates a response to casual factors. However, Bayesian regression embeds the functional form as the mean of a probabilistic distribution of the predictions and, incorporates the precision (squared inverse of the standard deviation) to capture uncertainty. Bayesian modeling normally employs the normal distribution for obtaining estimations for the mean response (or the expected value) of a stochastic variable. Assuming: (a) that one has a sample of  $n$  observations of a response (or a parameter)  $x_i$  with mean  $\bar{x}$ , (b) that this response (or parameter) is believed to follow a normal distribution with mean  $\theta$  and squared standard deviation  $\sigma^2$  and, (c) that the prior for each mean  $\theta$  is normally distributed  $N(\mu, \sigma^2/n_0)$  [normal with mean  $\mu$  and standard deviation  $\sigma^2/n_0$ ]. Then if the standard deviation of the likelihood and the prior is the same, and the prior comes from a sample of size  $n_0$ , Equation 2 is said to estimate the mean posterior distribution of the corresponding response (or parameter) by mixing the information contained in the prior with the likelihood. The expressions contained in Equation 2 can be expanded for the case of different standard deviations and reformulated in a matrix form for several parameters. As noticed, the variance of the posterior is smaller, because it combines two sources of information. The mean is the product of the prior mean ( $n_0\mu$ ) plus the likelihood mean ( $n\bar{x}$ ), weighted by their relative sample size, from where the means were estimated. As  $n$  goes to infinity the estimated mean posterior distribution looks like the likelihood (i.e. effect of prior vanishes). As  $n_0$  tends to zero the prior variance becomes larger and the distribution becomes flatter which is called a non-informative prior (13).

$$\theta / x \sim N \left( \frac{n_0\mu + nx}{n_0 + n}, \frac{\sigma^2}{n_0 + n} \right) \quad [2]$$

It can be shown (13) that Bayesian predicted responses are centered on the posterior mean with variance equal to the sum of the posterior variance and the sample variance. Bayesian regression is therefore capable of estimating stochastic variables (i.e., the coefficients of the functional form). This concept of stochastic variables can be extended to include missing data. Bayesian regression requires the specification of a functional form that relates the response with the causal factors and a dataset containing observed causal factors, responses and indexes for every modeled level. It also needs the

specification of prior distributions for every stochastic node, the precision and the error term if any. Priors are normally based on expert criteria or independent studies (informative) or just widely specified to cover the practical space of every stochastic node. However, the specification of the prior has a significant impact in the prediction if the model is based on a reduced amount of observations. Bayesian regression also requires a starting point for any stochastic variable. Missing predictors should be specified with "NA" and should be assigned a stochastic node which in turn requires an initial estimate. Under normal conditions initial points can be at any level of practical values without affecting the prediction because their influence vanishes quickly as the observations take over.

Results from Bayesian regression can be visualized via point estimates for every stochastic node (i.e., mean, media, and values for the Confidence Interval). It is possible to examine the posterior distribution of any parameter in order to verify if it follows the assumed probabilistic distribution. Also, it is possible to fit a regression to the scatter of observations for any causal factor and the response. Model checking is performed by observing the MCMC error, chains history, and the Deviance Information Criterion (DIC) which balance model complexity and goodness of fit (13).

Major advantages of Bayesian regression are: (a) it estimates model parameters from prior (initial opinion) and the observations which is equivalent to performing a calibration to local conditions. This calibration is an improved one because it contains measures of variability in the coefficients quantifying the associated uncertainty of the model. (b) it produces a probabilistic distribution that represents the mean prediction and the enveloping curves for a given Confidence Interval. (c) it is capable of dealing with homogeneous groups without partitioning the data by generating posteriors of predictors at different levels and models for every group; and (d) it is capable of dealing with missing data; that is, modeling missing predictors from a stochastic node and estimating missing responses by interpolation.

## Application of Bayesian Modeling to Pavement Deterioration

A Bayesian regression model can be used to predict future values of any condition indicator at any level of the causal factors (predictors). A MCMC simulation is conducted to sample the space of the causal factors in order to obtain a good approximation of

their probabilistic distribution. The use of confidence intervals is a natural approach for representing certainty about predictions. Building a Bayesian regression model involves: (a) identifying a functional model that captures the relationships between affecting factors and the response of interest (IRI, rut depth, deflections, distresses, etc.), (b) Setting up initial values in the form of prior distributions for the model coefficients, and (c) compiling some data (evidence) from observation or experimentation.

Step one requires the practitioner to borrow or develop a functional model that relates the response in terms of causal factors. The second step requires having initial values independent from the observed data (evidence). This can be approached in two ways: (a) an informative point of view where the opinions of experts are used to set up the initial values for the parameters or (b) results from external experiments or any other evidence is used for the initial estimates (i.e. a non-informative approach). The idea of using expert criteria for specifying the prior has been extensively debated (11, 12, 13 and 14). However, it is the only solution when no data is available from an independent source. Non-informative priors are preferred when there are sufficient observations.

#### Case Study – Rut depth progression Model

##### Revisiting Uzan (1983) Rut Depth Model

A prediction model of rut depth progression for a flexible (asphalt concrete) pavement section was developed by (10) from the AASHO Road Test. Section 581 of the AASHO Road Test (15) was used by (10) to calibrate a rut depth progression model. Section 581 is on lane 1 of Loop 4. Traffic loading for this section consisted of 6 vehicles operating 18 hours 40 minutes period each day, 6 days per week from November 5th 1958 to January 1960. After January 1960 traffic was increased to 10 vehicles, 7 days a week. Equivalent Single Axle Loads (ESALs) were computed based on the characteristics of the Road Test trucks provided in Table 7 of (15).

The model developed by (10) related the rate of rutting with traffic loading and pavement deflection under a moving standard dual wheel (single axle) for which pavement layer thickness and material properties were given. According to (10) the rate of rut depth per application relates to the traffic loading (N) by the following expression:

$$RR_t = \frac{pa}{E_s} k_1 N^{k_2} \quad [3]$$

where,  $RR_t$  = rate of rutting (in/year);  $p$  = contact pressure (psi);  $a$  = radius of contact of one side of a standard dual wheel axle (in);  $E_s$  = subgrade modulus (psi); and  $k_1$  and  $k_2$  = functions of the material properties and pavement geometry.

Similar models by AASHO (15), (16) and (6) support the assumption that rut depth is highly related to traffic loading. Uzan (10) used a second order polynomial expansion to express the values of  $k_1$  and  $k_2$  in terms of the pavement plastic properties for a four layer pavement structure. Equations 4 and 6 present a simplified form of those relationships considering that a sensitivity analysis conducted by (17) found that rut depth is strongly dependent on both: pavement deflection and plastic properties of the asphalt layer ( $\alpha$  and  $\mu$ ). Uzan (10) also found that plastic properties for the other granular layers (base and subbase) “vary over a small range...” and were “found to have little effect on the rut depth formation” (10). The transformed expressions contained on Equations 5 and 7 were needed for coding the software WinBUGS (13).

$$\log(1 + k_1) = 0.00694 + 0.027017 \times \log(1 + \mu_1) + 0.00235 \times \log(\alpha_1) \quad [4]$$

Which can be written,

$$k_1 = (1.01611 \times (1 + \mu_1)^{0.027017} \times (\alpha_1)^{0.00235}) - 1 \quad [5]$$

$$k_2 = -1.01024 + 0.217 \log(1 + \mu_1) - 1.26867 \times \log(\alpha_1) \quad [6]$$

Which can be written,

$$k_2 \times \ln(10) = \ln(10^{-1.01024}) + 0.217 \ln(1 + \mu_1) - 1.26867 \times \ln(\alpha_1) \quad [7]$$

where,  $\mu_1$  and  $\alpha_1$  are the plastic parameters for the asphalt layer.

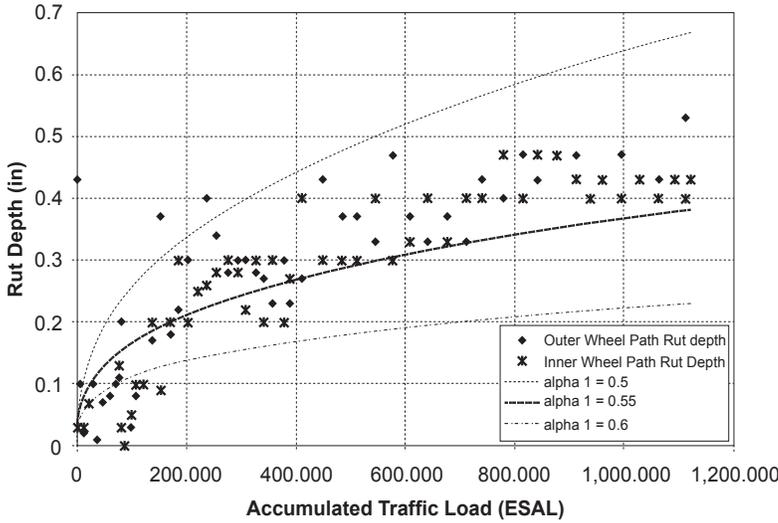
Uzan (10) noticed that the  $k_1$  coefficient is equal to zero when the pavement is elastic, i.e.  $\alpha_1 = 1$  and  $\mu_1 = 0$ . Equation 3 can be re-expressed to include the deflection under a moving standard dual wheel. Equation 8 presents such expression which alternatively can be transformed using logarithms as shown in Equation 9 below. Equation 10 expresses the total rut depth as a result of integrating Equation 9:

$$RR = \delta \times \frac{k_1}{W} \times N^{k_2} \quad [8]$$

Table 1 Parameters to Estimate Total Rut Depth

Plastic Properties	k1	k2	Equation for Total Ruta Depth
$\alpha_1 = 0.5$ and $\mu_1 = 0.3$	0.021659	-0.60356	Log RD = -0.865 + log $\delta$ + 0.3964 log N
$\alpha_1 = 0.55$ and $\mu_1 = 0.3$	0.021887	-0.65607	Log RD = -0.805 + log $\delta$ + 0.3493 log N
$\alpha_1 = 0.6$ and $\mu_1 = 0.3$	0.022096	-0.70402	Log RD = -0.729 + log $\delta$ + 0.2960 log N

Figure 1 Manual calibration of  $\alpha_1$  (alpha 1) by fitting the observed rut depth of section 581 of the AASHO Road Test.



$$\log(RR) = \log(\delta) + \log\left(\frac{k_1}{W}\right) + k_2 \log(N) \quad [9]$$

$$RD = \delta \times \frac{k_1}{W(1+k_2)} \times N^{1+k_2} \quad [10]$$

where,  $W =$  ;  $k_1$  and  $k_2$  are as defined under Equations 5 and 7;  $N$  = number of load applications (ESAL); and  $\delta$  = deflection under a moving standard load (inch).

However, (10) assumes  $\mu_1$  to be held constant at a level of 0.3, while, three curves were produced for values of  $\alpha_1$  ranging from 0.5 to 0.6 in order to calibrate the rut depth equation to those conditions observed during the AASHO Road Test (15). Table 1 present the fixed parameters employed to produce each of the calibration curves presented on Figure 1. Uzan (10) concluded that  $\alpha_1 = 0.53$  produced the best fit to the observed data.

#### Bayesian Regression Modeling of the AASHO Road Test Data

The model developed by (10) was selected to validate this paper's proposed approach. The WinBUGS free

software suite developed by the Medical Research Council of the Cambridge University was used for this purpose.

In the classical regression approach presented above, the values of the coefficients  $\alpha_1 = 0.53$  and  $\mu_1 = 0.3$  were assumed after model calibration. The pavement deflections were fixed at a level of 0.2 inches (5mm) on the model and they did not provide any information on reliability (or dispersion of the predictions). The Bayesian model proposed here assumes those coefficients to be stochastic variables and estimates a posterior probabilistic distribution from a combination of expert criteria and the observed data. For the validation of the Bayesian model the pavement layers was held constant with  $D_1 = 5$  inches (127mm),  $D_2 = 6$  inches (152.4mm) and  $D_3 = 12$  inches (304.8mm).

On the second part of this paper explicit consideration of the road resistance to rut depth was given by considering the thickness of the granular layers. According to (10) rut depth has a strong dependence on the coefficient  $\alpha_1$  and  $\alpha_1$  on the temperature. As seen on the calibration curves for  $\alpha_1$  on Figure 1, the larger the value of  $\alpha_1$  the lower the level of rut depth deterioration. Hence larger values of  $\alpha_1$  should be related to winter observations because low temperatures increase the bearing capacity of the soil and granular bases reducing rut depth damage. On the contrary, smaller values of alpha 1 should be related to spring observations as excess in moisture reduce the bearing capacity of the pavement structure. The coefficient  $\alpha_1$  can be used for a multilevel modeling on further applications of the modeling as presented here.

Any Bayesian Regression modeling requires prior information for the stochastic nodes (variables). The use of non-informative priors is suggested elsewhere (8, 13). Using expert criteria as prior information has advantages and disadvantages. In situations where few observations are available, expert criteria has a strong influence in the model posteriors and therefore in the prediction. On the contrary, for models where large datasets are available, the effect of prior information vanishes because the model is able to learn from the observations and correct any bias contained in the priors. Prior distributions for the model in the present study contained expert criteria on nodes  $\alpha_1$ ,  $\mu_1$ , the deflection  $\delta$  and the overall model precision  $\tau$  (see Figure 2 for specifics on codes and specified priors). The simulation was divided in two steps: The first part (known as the "burn-in") consisted of 20 thousand iterations conducted until convergence. The second part

consisted of 20 thousand more iterations and its results produced the posterior distributions of the parameters and the predictive model for rut depth.

The classical functional form provided by Equation 10 is used to represent the mean,  $E\{*\}$  accompanied by a variance term,  $VAR\{*\}$ . Both elements are embedded in a probabilistic distribution (normal in this case study). Equation 11 shows the specification of the regression model embedded in a normal distribution. The dependency of such expression on  $k_1$  and  $k_2$  is guaranteed using Equations 5 and 7.

$$RD = Normal\left(\delta \times \frac{k_1}{W(1+k_2)} N^{1+k_2}, \tau\right) \quad [11]$$

where,  $\delta \times \frac{k_1}{W(1+k_2)} N^{1+k_2}$  = mean of total rut depth in inches;  
 $\delta$  = deflection (inch);  $k_1$ ,  $k_2$  = simplified functions of material properties;  $\alpha_1$  and  $\mu_1$  are as defined in Equations 5 and 7;  $W = \frac{\delta \times E_s}{pa}$  is dimensionless deflection coefficient related to  $E_s$  = the modulus of elasticity of the subgrade;  $p$  = contact pressure of the tire and;  $a$  = the radius of contact of one side of a standard dual wheel axle; and  $N$  = number of load applications (ESAL). After validating the proposed approach, the entire dataset of rut depth observations was used to build a performance deterioration model. Pavements were clustered in two groups reflecting differences in the thickness of the base. A Bayesian regression model nested per base strength was used to calibrate the pavement parameters  $\alpha_1$  and  $\mu_1$  and to produce performance models for each one.

### Results

Uzan (10) calibrates the rut depth prediction model employing the classical fitting technique of least squares approach. He assumed  $\mu_1$  to be fixed at 0.3 and the deflection  $\delta$  to be fixed at 0.25 meanwhile, he manually produced curves for  $\alpha_1$  to calibrate the model obtaining the best fit to the observations. In our approach a initial Bayesian regression modeling for section 581 estimated that the mean value of  $\mu_1$  is 0.2856 varying for a 95% confidence interval between 0.1876 and 0.382, the deflection  $\delta$  was held constant at 0.25, and the mean estimate of  $\alpha_1$  laid very close to the traditional one (0.5305) with values for the 95% confidence interval ranging from 0.513 to 0.547. Therefore the Bayesian regression approach was validated. A Performance model for section 581 is shown in Figure 3, note the very broad range of variability (higher uncertainty) and expected performance similar to that observed in Figure 1. The next logical step was to develop models of rut depth progression per pavement's strength (based on

WinBUGS software code used for the validation analysis.

Figure 2

```

model {
  for (i in 1:56) {
    RD [i] ~ dnorm(mu[i],tau) # Number of observations section 581
    mu[i] <- (a1[i]/W[i]*(1+a2[i]))*delta[z[i]]*pow(N[i]+a2[i]) # mu = rut depth after Uzan (1983)
    a1[i] <- (1.01611*pow(1+u1,0.027017)*pow(alpha1,0.00235))-1 #Specification of Mean
    a2[i] <- (-2.32616+0.217*log(1+u1)-1.26867*log(alpha1))/2.302585
    W[i] <- (-20*delta[z[i]])
    res[i] <- (RD[i]-mu[i]/sigma) #standard residual
    p.res [i] <- phi(res[i]) #accumulated std residual
    Y.pred[i] ~ dnorm(mu[i],tau)
  }

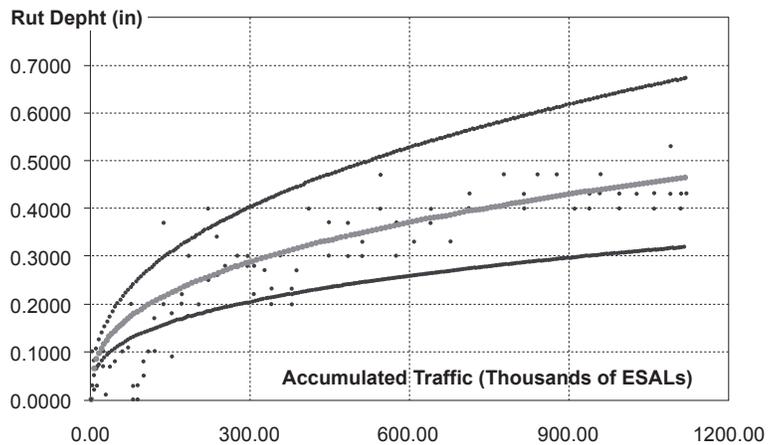
  for(r in 1:4) {
    delta[r] ~ dnorm(0.025,15000) #Prior - Expert Criteria
  }
  ul ~ dnorm(0.3,400) #Prior - Expert Criteria
  alpha1 ~ dnorm(0.5,100) #Prior - Expert Criteria
  tau ~ dgamma(0.0001,0.0001)
  sigma <- 1/sqrt(tau)
}

list(delta=c(0.035,0.035,0.035,0.015), alpha1 = 0.6, #Initial Values Chain 1
u1 = 0.25, tau = 0.0001)
list(delta = c(0.015,0.015,0.015,0.015), alpha1 = 0.42, #Initial Values Chain 2
u1 = 0.35,tau = 0.0001)

```

Bayesian Performance model for section 581.

Figure 3



thickness of the base). For this case Bayesian regression models estimated that the mean value of  $\mu_1$  was 0.989 varying for a 95% confidence interval between 0.912 and 1.07, the deflection was eliminated from this models as this factor cancels out in the functional form (Equation 11) and, the mean estimate of  $\alpha_1$  reached 0.568 for pavements with weak base and 0.583 for strong based pavements with tight variation (as compared to Figure 4) in both cases for the 95% confidence interval, ranging from 0.5607 to 0.5757 and from 0.5752 to 0.5912.

Figure 4 shows observed differences in the parameter alpha1 for both clusters. One can conclude that there are

Figure 4 Bayesian performance models per pavement's base strength.

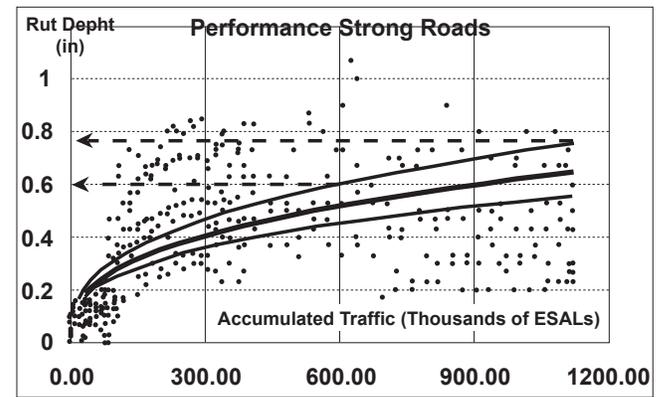
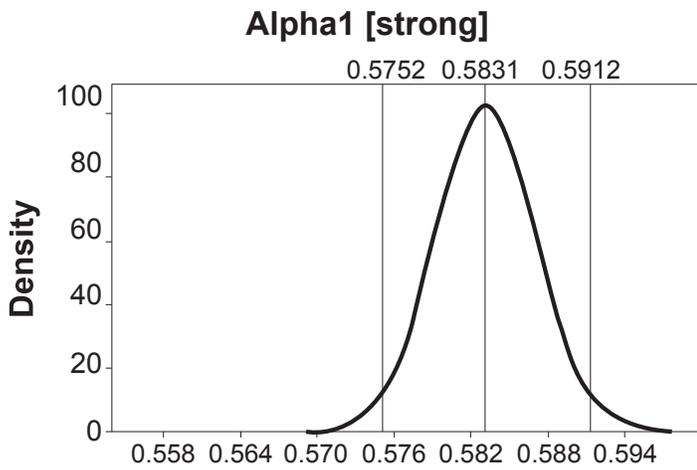
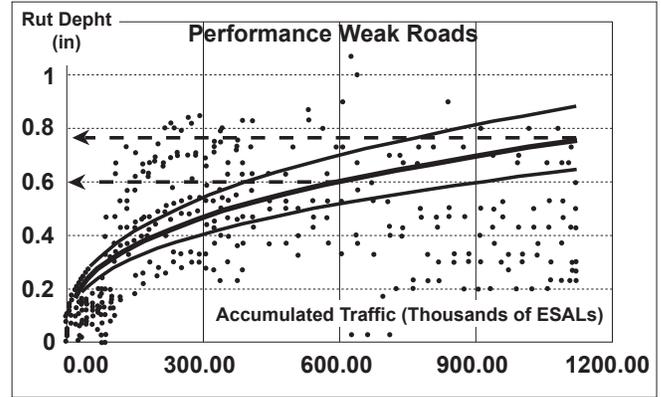
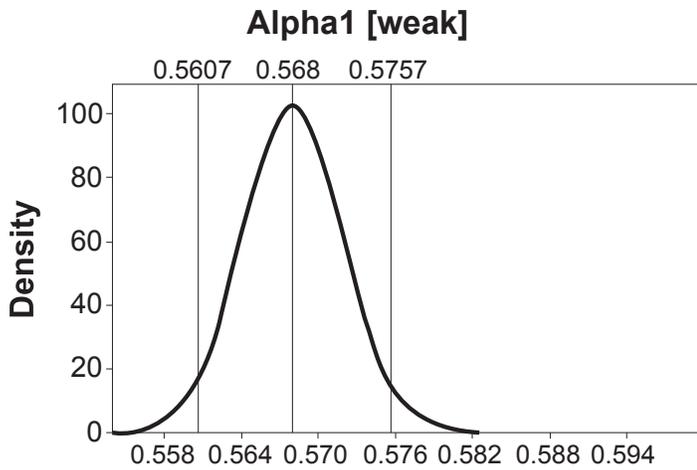


Table 2 Transition Probabilities Matrix for pavements with thicker base

0.0 0.3 0.3 0.3 0.4 0.4 0.4 0.4 0.4 0.5 0.5 0.5 0.5 0.5 0.6 0.6 0.6 0.6 0.6 0.7 0.7 0.7  
 9 - 5 - 7 - 9 - 1 - 3 - 5 - 7 - 9 - 1 - 3 - 5 - 7 - 9 - 1 - 3 - 5 - 7 - 9 - 1 - 3 - 5 -  
 0.1 0.3 0.3 0.4 0.4 0.4 0.4 0.5 0.5 0.5 0.5 0.5 0.6 0.6 0.6 0.6 0.6 0.7 0.7 0.7 0.7  
 1 7 9 1 3 5 7 9 1 3 5 7 9 1 3 5 7 9 1 3 5 7

10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	12	18	26	24	14	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	3	6	13	19	20	17	12	6	3	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	2	4	8	13	16	18	16	11	7	3	2	0	0	0
0	0	0	0	0	0	0	0	0	1	2	3	7	11	14	16	15	12	9	5	3	1

Note: 1 inch = 25.4mm

significant differences by observing how the estimated distribution of alpha1 for stronger pavements is shifted approximately two standard deviations towards the right of the same measure for weaker roads. The

corresponding performance model of each group of pavements is given on the right hand side of same figure. While strong pavements are expected to reach 0.5 inches (12.7mm) and 0.65 inches (16.5mm) of rut depth at 600,000 and 1,120,000 ESALs respectively, weaker pavements would be reaching 0.6 inches (15.2mm) and almost 0.8 inches (20.32mm) at the same levels of accumulated traffic. Expected rut depth deterioration -originally observed for 2 years at the AASHO road test- will keep increasing with additional accumulated traffic across the lifespan of any road. Figure 4 explicitly captured the observed variability of both parameters and the overall performance. The mean prediction -shown as a solid gray line- is accompanied with an envelope for the 95% confidence interval (black lines).The results of Figure 4 can be re-expressed into a Transition Probability Matrix (TPM). Table 2 shows a TPM derived for the strong based pavements of Figure 4.

## Conclusions

This paper has demonstrated how Bayesian Regression modeling provides a more reliable framework for prediction. The framework includes measures of variability on both the posterior distribution of the predictors and the overall prediction for any desired confidence interval.

The paper has validated the Bayesian regression modeling by comparing the results with those from a classical regression model developed from the AASHO Road Test. The mean of the response of the model parameters from the Bayesian regression model reproduce those of the classical approach very closely.

The model presented in this paper is capable of estimating several stochastic parameters at once. Therefore, the method is very useful as a tool to calibrate models to local conditions from the observed data of the causal factors and response. Estimations are accompanied by a confidence interval which establishes the level of reliability of the model. Practical development of a transition probability matrix has also been demonstrated from final results of the Bayesian regression model.

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