UNDERGRADUATE STUDENTS' MODELLING ROUTES MEDIATED BY TECHNOLOGY IN THE LEARNING OF LINEAR TRANSFORMATIONS

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Technology-mediated mathematical modelling has been recognized as a significant learning context. This paper reports on a study aiming to characterize the modelling routes performed by Costa Rican undergraduate students when solving a mathematical modelling task using technological resources as a means to explore the concept of linear transformation. Data collected from audio records of students' discussions, their written work, and GeoGebra files show that the group of two students we focus in this paper performed a linear modelling route, by creating a mathematical model with the software, which highlights the visual aspect and the geometric transformation concept.

INTRODUCTION

The learning of Linear Algebra needs to be revitalized through innovative teaching proposals involving contextualized situations, namely mathematical modelling contexts (Blum & Borromeo Ferri, 2009). In fact, studies developed in such learning contexts where technology resources were not used reveal undergraduate students' difficulties in understanding the concept of linear transformation, both in connecting it to particular functions and in realizing the meaning of linear transformation from the point of view of a geometric representation (Trigueros & Bianchini, 2016). Taking into account that few studies address technology as a fundamental resource in modelling tasks (Greefrath, Hertleif, & Siller, 2018) and that some studies, such as Borromeo's (2007), investigate modelling routes from a cognitive perspective without a focus on technological resources, it is relevant to investigate the students' routes in mathematical modelling tasks using technology. Thus, this study aims to characterize the modelling routes performed by Costa Rican undergraduate students attending a Linear Algebra course when solving a mathematical modelling task involving the concept of linear transformation, with GeoGebra. Specifically, we present and discuss here the modelling route of a group of two students on a modelling task concerning 2D image scaling.

MODELLING ROUTES WITH TECHNOLOGICAL RESOURCES

The process of mathematical modelling entails translations in both directions: from the real world to mathematics and vice-versa (Borromeo, 2018; Blum & Borromeo, 2009). This process is usually not linear but rather different students perform different paths or modelling routes (Borromeo, 2018).

In investigating students' modelling from a cognitive perspective, the focus is centered on the thinking processes that are followed by the students during their modelling process (Borromeo, 2007), which may be represented and depicted through a modelling route. In simple terms,

> The individual starts this process during a certain phase, according to his or her preferences, and then goes through different phases several times or only once, focusing on certain phases and/or ignoring others. (p. 265)

During the course of a modelling route the student is supposed to develop certain activities associated with an ideal modelling cycle: (1) understand the task; (2) simplify and structure the task; (3) mathematize the real model; (4) work mathematically on the model; (5) interpret results; (6) validate results in the context of the actual situation; and (7) respond to the problem situation by presenting the results. It is possible for the student to go through all or only some of these activities, depending on factors such as their level of mathematical competencies, mathematical thinking styles, and level of mathematical knowledge and own experiences (Borromeo, 2018).

In modelling activities, technology becomes a resource that facilitates visualization, exploration, organization and inspection of large sets of data, and therefore the integration of technology in the modelling cycle can represent an extension of the modeling process (Siller & Greefrath, 2010, p. 2137). In particular, as long as the task motivates making use of technological tools, dynamic software such as GeoGebra can be a powerful resource to support the modeling work developed by the student (Greefrath et al., 2018).

CONTEXT AND METHODOLOGY

The proposed modeling task involves a 2D image scaling context, where students have to find a model that allows them to convert the size of a photograph of the Big Ben into the scale defined by another photograph (fig. 2). Students worked autonomously in groups of 2 or 3 individuals in a computer laboratory where they had access to software such as GeoGebra, Mathematica, and Excel. During the work on the task, the researcher, who also acted as the teacher (first author), participated in some of the discussions in the working groups. The participants were 21 students enrolled in a Linear Algebra course during the first semester of 2019, the majority being engineering students. The study has a qualitative and interpretative nature (Cohen, Manion & Mohinson, 2007). Data collection included: participant observation, with audio recording of the students' discussions in some of the ten groups in which the class was divided; the students' written work on the proposed task; and their digital GeoGebra files. The data analysis makes use of the modeling cycle proposed by Greefrath et al. (2018), featuring in particular the modeling routes followed by a group of two students, Diogo and Henrique (fictitious names).

RESULTS

In solving the modelling task, the use of GeoGebra has shown to be significant for Diogo and Henrique, by allowing them, unlike other groups that did not use technology, to work on the mathematical model and interpret mathematical results. Diogo and Henrique's modelling route is represented in figure 1, through the sequence of red arrows that indicate a rather complete and straightforward modeling route. It reveals that the students crossed the world of mathematics using the technological resource, and returned again to the real world situation with a solution to the real problem of scaling the photo of the Big Ben.

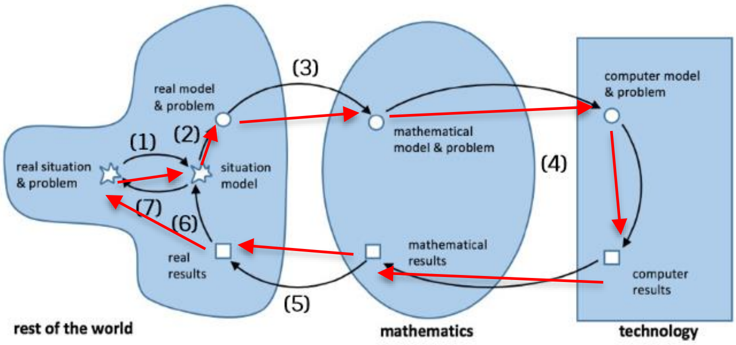

Figure 1: The modelling route of Diogo and Henrique.

Initially, Diogo and Henrique associated the real situation to the problem of enlarging the magnitude of a vector. This led to the idea that the "zoom" could be defined by the ratio between the magnitudes of two vectors in the plane (1). Then they decided to define vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ to measure the width of the Big Ben's face in each of the images provided (2). This allowed creating a real model, as it is evidenced in the following dialogue, where Diogo and Henrique associated the magnitudes of the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ to lengths of homologous sides in similar figures.

Professor: So, what did you notice about the relationship between the photos?

Henrique: We are looking at the distance from here to there (pointing to the two points on the smaller scale image) and at the distance from here to there (pointing to the homologous points on the larger scale image), then here we are observing an enlarging factor.

To build the mathematical model, Diogo and Henrique mathematized the real model by deciding to find out how many times would $\overrightarrow{AB}$ fit in $\overrightarrow{CD}$, and thus defining the value $\frac{CD}{AB}$ as the dilation factor associated to the "zoom in" performed (3). To obtain a mathematical result that would provide the scale factor, Diogo and Henrique resorted to GeoGebra (4), where they inserted the images given in the task statement and then defined the two pairs of points A, B and C, D as shown in figure 2.

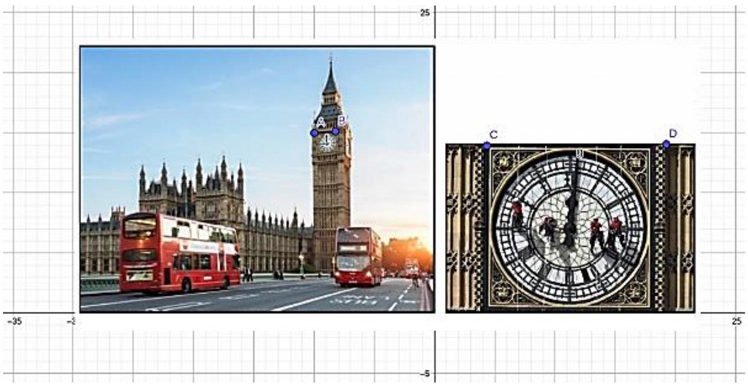

Figure 2: Approach to the task in GeoGebra by Diogo and Henrique

Working on Geogebra, Diogo and Henrique got the values $AB = 1{,}77$ and $CD = 14{,}90$ and thus obtained the mathematical result 8,41 for the scale factorCD/AB, which was taken as the magnifying factor between the images (5). The students were able to interpret this value when stating that "to make $\overrightarrow{AB}$ look like the size of $\overrightarrow{CD}$, the value of the zoom should be 8,41" (6). After that, Diogo and Henrique answered to the problem by mentioning that the enlargement of the image was the result of applying the scale factor found, although they did not go through the process of validating their model (7), namely by performing the dilation on the smaller image.

More specific details about Diogo and Henrique's mathematical modelling process, including excerpts from their written work and oral discussions, are available and support the broader analysis of their modelling route.

CONCLUSION

The results show that the modelling route followed by Diogo and Henrique tends to be rather straightforward and it is characterized by performing steps of the modelling cycle that cross the three worlds: the real world, the mathematical world, and the technological world. The use of GeoGebra mediated their modelling process as they turned to the software to objectify their mathematical model of a scaling operator on IR^2, using the idea of computing images of vectors under a dilation. This is consistent with a geometric transformation, thus evidencing the contribute of the software as a crucial component of the modelling activity (Greefrath et al., 2018).

From the data, it was evident that the algebraic concept of linear transformation was not used by the students, namely the corresponding matrix transformation. Instead, they kept working on the coordinate system and remained with the visual geometrical transformation of vector dilation. This could be associated with the use of the GeoGebra tools that the student chose to work with, possibly because those were familiar to them. Finally, the lack of validation of the scaling factor found by Diogo and Henrique on the actual images suggests the need to make students aware of the role of validating models as a way of both testing and improving an early model.

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