Estimation of the yield curve for Costa Rica using metaheuristic optimization

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Abstract

The term structure of interest rates or yield curve is a function relating the interest rate with its own term. Nonlinear regression models of Nelson-Siegel and Svensson were used to estimate the yield curve using a sample of historical data supplied by the National Stock Exchange of Costa Rica. The optimization problem involved in the estimation process of model parameters is addressed by the use of metaheuristics: Ant colony, Genetic algorithm, Particle swarm and Simulated annealing. The aim of the study is to improve the local minimum obtained by an optimization method using descent direction. Good results with at least two metaheuristics are achieved, Particle swarm and Simulated annealing.

Keywords: Yield curve, Nelson-Siegel model, Svensson model, Ant colony, Genetic algorithm, Particle swarm, Simulated annealing.

1 Introduction

The interest rate is essential in the modern economy, it refers to the payment of money from a debtor to a creditor by use of capital [11]. There are many factors that determine the level of interest rates: inflation risk, uncertainty, quality of information, random fluctuations and the period of investment, among others. Remaining constant all factors affecting the level of interest rates, except the period of investment is called term structure of rates interest [11].

In a technical document by Bank for International Settlements [3] presented methodologies and models used by 13 nations in the estimation of

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the yield curve, which highlight the parametric models of Nelson-Siegel and Svensson.

One of the papers, which is an important reference, is presented by the Central Bank of Canada [4]. The paper introduces the parametric models of Nelson-Siegel and Svensson for estimating the yield curve in the Central Bank of Canada. The optimization problem was faced with two methods called partial-estimation algorithm and full-estimation algorithm. It was concluded that the optimization process can be improved. Moreover, given the large size of the search space, genetic algorithms was suggested as a method that can improve the estimation.

The stock market in Costa Rica is small, therefore many of the existing methods are not feasible to implement. The paper of Barboza et al. [2] mentions that after reviewing existing models to estimate the curve, the most suitable for the Costa Rica market is the Svensson model. It also proposes a modification to the objective function, in order to consider topics such as historical data and volatility.

The optimization problem in the area of the yield curve for Costa Rica was worked by Piza et al. [22]. In that study numerical methods such as Gauss-Newton, gradient descent and Marquardt were used. It was concluded that a successful optimization depends on the initial values and also indicated that only local minimum were obtained. It is recommended the use of Metaheuristics to address the problem of finding the global minimum.

In the present paper, Metaheuristics are implemented to improve local minima that are achieved using methods that work with descent direction in the problem of estimating the parameters of the nonlinear regression models of Nelson-Siegel and Svensson.

2 Data

Historical data were provided by the Bolsa Nacional de Valores (BNV, National Stock Exchange). These are bonds and zero coupon bonds issued by the Central Bank and the Ministry of Finance Costa Rica, which are called tp0, tp, bem0 and bem.

Data are for the period of February 23, 2015 to March 12, 2015, only emissions in colones were considered and there is not any restriction on the amounts of transactions.

3 Yield curve estimation

The yield curve relates interest rates with its own term [9, 11], this rate is call spot interest rate.

The forward interest rate is an interest that is negotiated today for a transaction that will occur in the future [21]. The forward rate is an expectation of what the spot rate will be in the future [9].

If there are continuous rates δ_t y δ_s for terms t and s, $(s < t)$, it is defined the forward continuous rate as [2, 13]:

$$f_{t,s} = \frac{t\,\delta_t - s\,\delta_s}{t - s}.$$

The instantaneous forward rate is obtained as a limit [2, 13, 21]:

$$f_t = \lim_{s \to t} f_{t,s}.$$

The Nelson-Siegel model (1987) [18] proposes a continuous function to describe the shape of the instantaneous forward rate depending on the term t,

$$f_t = \beta_0 + \beta_1\, e^{-\lambda t} + \beta_2\, \lambda\, t\, e^{-\lambda t}. \tag{1}$$

From equation (1) a continuous function is obtained for the spot rate,

$$\delta_t = \beta_0 + \beta_1 \left(\frac{1 - e^{-\lambda t}}{\lambda t} \right) + \beta_2 \left(\frac{1 - e^{-\lambda t}}{\lambda t} - e^{-\lambda t} \right).$$

The Svensson model (1994) [27] extends the Nelson-Siegel model by incorporating two parameters more: β_3 y λ_2. Thus the continuous function for forward rate is,

$$f_t = \beta_0 + \beta_1 e^{-\lambda_1 t} + \beta_2\, \lambda_1\, t\, e^{-\lambda_1 t} + \beta_3\, \lambda_2\, t\, e^{-\lambda_2 t}, \tag{2}$$

and from (2) the function for the spot rate is

$$\delta_t = \beta_0 + \beta_1 \left(\frac{1 - e^{-\lambda_1 t}}{\lambda_1 t} \right) + \beta_2 \left(\frac{1 - e^{-\lambda_1 t}}{\lambda_1 t} - e^{-\lambda_1 t} \right) + \beta_3 \left(\frac{1 - e^{-\lambda_2 t}}{\lambda_2 t} - e^{-\lambda_2 t} \right).$$

If the spot rates for different maturities are available the price of a bond can be calculated as [9]

$$Pr = \sum_{k=1}^{t} c\, e^{-\delta_k\, k} + F\, e^{-\delta_t\, t}, \tag{3}$$

where c is the coupon and F is the face amount of the bond.

On the other hand, with a sample of bonds price the parameters of the Nelson-Siegel and Svensson models can be estimated. The estimation is obtained by minimizing the objective function (4) with respect to $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \lambda)$ parameters of Nelson-Siegel model or $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1, \lambda_2)$ parameters of Svensson model.

The objective function is given by the least square criterion with weighting factors proposed in [2]. These weighting factors allow using historical observations and it also reduces volatility through a stock measure.

$$\sum_{k=1}^{n} \frac{(Pr_k - \tilde{Pr}_k)^2}{H_k(1 + ND_k)}, \tag{4}$$

where Pr_k is the observed price for the bond k, $\tilde{Pr}_k$ is the estimate price for the bond k obtained by (3) as a function of $\boldsymbol{\theta}$,H_k is the bid-ask spread and ND_k is the number of days from the bond k was traded.

A set of constraints similar to those used in [4] have been implemented with two goals, results economically feasible and speed in the optimization process.

The following constraints are for the Nelson-Siegel model:

$$\begin{aligned} 0\% &< \beta_0 < 25\% \\ -20\% &< \beta_1 < 20\% \\ 0\% &< \beta_2 < 25\% \\ 1/300 &< \lambda < 12 \\ 0 &< \beta_0 + \beta_1. \end{aligned} \tag{5}$$

Constraints for the Svensson model:

$$\begin{aligned} 0\% &< \beta_0 < 25\% \\ -20\% &< \beta_1 < 0\% \\ 0\% &< \beta_2 < 25\% \\ 0\% &< \beta_3 < 25\% \\ 1/300 &< \lambda_1 < 12 \\ 1/300 &< \lambda_2 < 12 \\ 0 &< \beta_0 + \beta_1. \end{aligned} \tag{6}$$

4 Optimization methods

In order to minimize (4) the following metaheuristics were used: Genetic algorithm [6], Ant colony [5], Particle swarm [12] and Simulated annealing [1]. These metaheuristics were programmed

The results obtained with the metaheuristics were compared with the results of the Quasi-Newton algorithm BFGS applied through an adaptive barrier method [14]. To implement these methods the built-in R [23] functions *constrOptim* and *optim* were used.

Genetic algorithm: Algorithm based on ideas of genetic evolution and biology [7, 17]. It starts with a population of solutions chosen randomly, in each iteration a new population is obtained from the previous one by pairing, mating and mutation. Chromosomic representation is based on numerical vector of parameters (4 parameters for Nelson-Siegel model, 6 parameters for Svensson model).

In this work roulette wheel rank weighting is used as selecting pairing, 50% as natural selection, 10% as mutation. For mating it is used the algorithm based on [7], a mix of extrapolation method with crossover method.

Ant colony: Metaheuristic that takes its ideas from the way ants get food [5, 24, 25]. In this study it is used the version for continuous domains presented in [24]. The pheromones are used by means of an array that stores a number of solutions and new solutions are built sequentially using the information of the array.

From the work of Socha [25] it was taken the parameters of the method: 50 solutions store, 2 ants, 0.4 locality of the search and 1.1 speed of convergence.

Particle swarm: Based on the behavior of some groups of animals [20, 30]. The performance of an individual is influenced by its best historial performance and the best performance of the group up to the present iteration.

Taking into account the recommendation made by [8], 47 particles were used, -0.1832 of inertia, 0.5287 as cognitive parameter and 3.1913 as social parameter.

Simulated annealing: Based on the physical process named annealing, which takes a solid to a high temperature and then let it cool very slowly in order to get a more resistant and pure state of the solid [1, 15, 28]. Also it uses the Metropolis criterion of acceptation whose purpose is to get out of local minimum zone [15, 26, 28].

In this paper it is used the version named very fast simulated reannealing [10] which allowd to work with restrictions. The size of the Markov chain was established in 100, and the temperature is updated with the factor 0.95.

BFGS Quasi-Newton algorithm: This algorithm is a Line Search method [19] where the search direction is given by a modified Newton direction. An approximation to the Hessian is used and it is updated in each iteration [19].

An adaptive barrier method: The objective function depends on the vector $\boldsymbol{\theta}$ which has to satisfy (6) or (5), so that the constrained optimization problem in changed into an unconstrained problem, and an adaptive barrier method is used [16].

In this case a logarithmic barrier is added to the objective function in order to handle the constraints (6) or (5). If the minimum lies on the boundary the barrier will not allow to reach it, to deal with this the logarithmic barrier has a component that changes in each iteration [14].

5 Results

For each method a multistart strategy [29] of size 2,000 was made. The way of comparison is as follows: the best objective function value for the metaheuristics is the expected value from their multistart, in the case of the adaptive barrier the best objective function value is the minimum value that was achieved from its multistart.

The following values are reported: the objective function value, the coefficient of variation information taken from the multistart, the goodness of fit and the average time of running the R-code measured in seconds.

Table 1: *Summary metaheuristics performance in estimating the Nelson-Siegel model.*

Algorithm	Objective function value	Coefficient of variation	Goodness of fit	Average time (m)
Particle S.	441.5018	<1%	0.003345%	00:22
Simulated A.	441.5034	<1%	0.003353%	00:28
Adaptive barrier	441.5243	240%	0.003354%	—
Genetic A.	1,206.0571	18%	0.066502%	01:16
Ant C.	1,207.6136	36%	0.066541%	00:18

The results for the Nelson-Siegel model are shown in the Table 1. Two metaheuristics got better results than the adaptive barrier method, namely, Particle swarm and Simulated annealing. Their coefficient of variation are almost zero indicating that the same results are obtained almost every time the functions are run. The average time is approximately 20 seconds.

Table 2: *Summary metaheuristics performance in estimating the Svensson model.*

Algorithm	**Objective function value**	**Coefficient of variation**	**Goodness of fit**	**Average time (m)**
Particle S.	251.5805	<1%	0.012147%	00:43
Simulated A.	251.6899	<1%	0.012550%	00:46
Ant C.	254.6444	84%	0.012345%	01:32
Adaptive barrier	441.6267	317%	0.003164%	—
Genetic A.	1,138.3852	12%	0.052407%	00:13

In the case of the Svensson model three metaheuristics had better performance: Particle swarm, Simulated annealing and Ant colony. But we highlight Particle swarm and Simulated annealing which have a coefficient of variation almost zero and an average time of 40 seconds.

6 Concluding remarks and further research

Two metaheuristics were better in both models, Particle swarm and Simulated annealing. These metaheuristcs besides of having the best results, their algorithms are easy to implemented, the execution time is acceptable and the outcomes are very stable.

Particle swarm and Simulated annealing are recommended for getting the parameters of the Nelson-Siegel and Svensson models.

For future research it is suggested to repeat this study with other sets of sample data so as to confirm the results obtained so far. For another data set, similar restrictions for (5) or (6) which are adjusted to the Costa Rican market characteristics, should be determined. Finally, a review of the configuration used in Genetic algorithm and Ant colony could also be made in order to obtain satisfactory parameters that could make compete these metaheuristics with the better ones.

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